1. It follows from the work on page 1, chapter 3 of the lecture notes that
\[
\int_{-\infty}^{\infty} e^{-\lambda y^2} \cos(2\lambda x_0 y) \, dy = e^{-\lambda x_0^2} \left(\frac{\pi}{\lambda}\right)^{1/2} \text{ for all } x_0 \in \mathbb{R} \text{ and all } \lambda > 0.
\]

The method there used Cauchy’s theorem to shift the path of integration. Now find an essentially different method of proof.

3. Use the method of steepest descent to find the first three terms in the asymptotic expansion of
\[
\int_C \frac{e^{\lambda(z^2-1)}}{z - 1/2} \, dz \text{ as } \lambda \to \infty,
\]
where \(C\) is the vertical line from \(z = 1 - i\infty\) to \(z = 1 + i\infty\).

Note that the process of deforming \(C\) into the steepest descent path yields a residue term due to the pole at \(z = \frac{1}{2}\).

4. page 69 / 3 \((\text{Bi}(\lambda) \text{ as } \lambda \to \infty)\)

5. page 69 / 4 \((\text{Ai}(-\lambda), \text{Bi}(-\lambda) \text{ as } \lambda \to \infty)\)

In the last problem also draw a plot of the \(z\)-plane, indicate the saddle points and the paths of steepest descent of \(\phi\), and shade the regions in which \(\phi > \phi_0\). Explain how the curves \(C_1, C_2, C_3\) are deformed so that Laplace’s method can be applied.

Note that the asymptotic properties of \(\text{Ai}(\lambda)\) and \(\text{Bi}(\lambda)\) for \(\lambda \to \pm \infty\) will be used later when we study turning points.