In the problems below, find the first two terms in the asymptotic expansion of \( w(z) \) as \( z \to \infty \). These are 2nd order equations, so there are two independent solutions; write them in the form \( w(z) \sim e^{\lambda z} z^\sigma (\alpha_0 + \frac{\alpha_1}{z} + \cdots) \) as \( z \to \infty \); you may take \( \alpha_0 = 1 \).

1. page 110 / 1 (i), (ii) : use the expansion \( w(z) \sim e^{\lambda z} z^\sigma (\alpha_0 + \frac{\alpha_1}{z} + \cdots) \) as \( z \to \infty \)

Hint for 1(i): note that (6.13) does not hold. Use the transformation following (6.20) in the form \( z = t^2, w(z) = t^{1/2} u(t) \) to derive an equation for \( u(t) \) for which (6.13) does hold and then apply the form (6.14).

Hint 1 for 1(ii): eliminate the first derivative term and apply (6.14).

Hint 2 for 1(ii): show that one of the two asymptotic solutions is exact

2. page 110 / 2 (i), (ii) : use the expansion \( w(z) \sim \exp(\phi_0(z) + \phi_1(z) + \cdots) \) as \( z \to \infty \)