Math 571 - Winter 2006
Numerical Methods for Scientific Computing I
Numerical Linear Algebra

Time: MWF 10-11am, 4088 East Hall

Instructor: Robert Krasny, 4830 East Hall, 763-3505, krasny@umich.edu

Office Hours: MWF 11am-12noon and by appointment or just drop in

Website: http://www.math.lsa.umich.edu/~krasny/math571.html

Text: *Numerical Linear Algebra*, by L.N. Trefethen and D. Bau, SIAM

From the preface to the text: "We hope the reader will come to share our view that if any other mathematical topic is as fundamental to the mathematical sciences as calculus and differential equations, it is numerical linear algebra."

Math 571 is an introduction to numerical linear algebra, a core subject in scientific computing. Three types of problems are considered: (1) solving a system of linear equations ($Ax{=}b$), (2) computing eigenvalues and eigenvectors of a matrix ($Ax{=}\lambda x$), and (3) least squares problems ($\min ||Ax - b||_2$). These problems arise in applications in science and engineering, and many methods have been developed for their solution, but standard methods may fail if the problem is large or ill-conditioned, e.g. the operation count may be prohibitive or computer roundoff may ruin the answer. We'll investigate these issues and study some of the accurate, efficient, and stable methods that have been devised to overcome these difficulties.

Topics: (1) vector and matrix norms, orthogonal matrices, projectors, singular value decomposition (SVD); (2) least squares problems, QR factorization, normal equations, Gram-Schmidt orthogonalization, Householder triangularization; (3) stability, condition number, IEEE floating point arithmetic, backward error analysis; (4) direct methods for $Ax = b$, Gaussian elimination, LU factorization, pivoting, Cholesky factorization; (5) eigenvalues and eigenvectors, Schur factorization, reduction to Hessenberg and tridiagonal form, power method, inverse iteration, shifts, Rayleigh quotient iteration, QR algorithm; (6) iterative methods for $Ax = b$, Krylov methods, Arnoldi iteration, GMRES, conjugate gradient method, preconditioning; (7) applications: image compression by SVD, finite-difference scheme for a two-point boundary value problem, Dirichlet problem for the Laplace equation, least squares data fitting

Prerequisites: a course in linear algebra (e.g. Math 217, 417, 419, 513, or equivalent); computing will be required on some homework - Matlab is recommended

Grading Policy: homework = 30%, midterm exam = 30%, final exam = 40%

Final Exam Date: Wednesday, April 26, 10:30am-12:30pm