hw#1 , due: Monday, January 23

0. Give a brief description of your academic background and research interests. If you work in a lab or research group, give your supervisor's name and describe your project. One paragraph is fine.

page 15 (orthogonal vectors and matrices) 2.2a , 2.3 , 2.6

page 24 (norms) 3.1 , 3.2

1. a) Let λ be an eigenvalue of an orthogonal matrix. Show that $|\lambda| = 1$.

b) A permutation matrix is a square matrix whose elements are either 0 or 1, such that every row and every column has exactly one nonzero element. (Such a matrix arises by permuting the rows or the columns of the identity matrix.) Show that a permutation matrix is orthogonal and hence its eigenvalues satisfy $|\lambda| = 1$. Give an example of a 4×4 permutation matrix whose eigenvalues are $\lambda = \{\pm 1,\ \pm i\}$.

c) Consider the rotation matrix, $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Show that R_θ is orthogonal and find its eigenvalues.

2. Let $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Solve $Ax = b$ by the spectral method.

3. Prove the following statements.

a) Given a vector norm $||x||$, the formula $||A|| = \sup\limits_{x \neq 0} \dfrac{||Ax||}{||x||}$ defines a matrix norm. (This is the induced matrix norm corresponding to the given vector norm.)

b) $||A||_\infty = \max\limits_i \sum\limits_j |a_{ij}|$ (hint: make sure you understand the derivation of the analogous formula for $||A||_1$ that was given in class.)

4. Consider the 2-point BVP, $-y'' + (4x^2+2)y = 2x(1+2x^2)$, $y(0) = 1$, $y(1) = 1+e$. Show that $y(x) = x + \exp(x^2)$ is the exact solution. Write a Matlab code to solve the problem using the 2nd order finite-difference scheme discussed in class, $-D_+D_-u_i + c_iu_i = f_i$. Use meshsize $h = 1/2^p$, where p is a positive integer. Solve the linear system using the tridiagonal LU method derived in class (you get partial credit if you use a full matrix; to get full credit you should use only vectors). Turn in the program listing. For $p = 1:4$, plot the exact solution ($y(x)$ vs. x) and the numerical solution (u_i vs. x_i, including the boundary points). For $p = 1:20$, present a table with the following data. column 1: h; column 2: $||u_h - y_h||_\infty$; column 3: $||u_h - y_h||_\infty / h^2$; column 4: cpu time; column 5: (cpu time)$/n$ (where $h = 1/(n+1)$ as in class). Explain the trends in each column.
Hints:
a) Debug your code using small values of p, e.g. $p \leq 10$; otherwise the cputime is too large.
b) Type `help cputime` in Matlab to learn how to find the cpu time.
c) Try different output formats, e.g. `format long`, `format short e`, `format short g`.