hw3 due: Friday , February 24

page 47 (projectors) 6.4 , 6.5

page 55 (QR factorization) 7.3

page 68 (Matlab) 9.2 , 9.3

extra instructions/hints for 9.3:

i. in part (c), plot the results in a $3 \times 4$ matrix using `subplot`

ii. use the following commands to display an image of the matrix $B$

```
map = [1.  1.  1.; .75 .75 .75; .5 .5 .5; .25 .25 .25; 0 0 0];
colormap(map)
pcolor(B)
```

iii. `pcolor` flips the matrix; you need to unflip it

1. In hw2, you showed that if $||x||$ is a vector norm and $W$ is a nonsingular matrix, then the expression $||x||_W = ||Wx||$ defines a new vector norm. Let $||A||$ denote the matrix norm induced by the original vector norm $||x||$ and let $||A||_W$ denote the matrix norm induced by the new vector norm $||x||_W$. Show that $||A||_W = ||WAW^{-1}||$.

2. Let $A \in \mathbb{C}^{m\times n}$, $m \geq n$, rank $A = n$.

a) Show that $A^*A$ is invertible.

b) Let $P = A(A^*A)^{-1}A^*$. Show that $P^2 = P$, $P^* = P$, and range$P$ = range$A$. (This shows that $P$ is the orthogonal projector onto range$A$.)

c) Show that $P = A(A^*A)^{-1}A^*$ reduces to $P = \hat{Q}\hat{Q}^*$, where $A = \hat{Q}\hat{R}$ is the reduced $QR$ factorization of $A$.

3.a) Let $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$. In class we applied the classical Gram-Schmidt method to compute the reduced QR factorization of $A$. Now apply the modified Gram-Schmidt method and Householder's method to compute the reduced QR factorization of $A$. You may check your answers in Matlab, but work out the steps by hand.

b) Find the orthogonal projector onto range$A$.

4. Let $A \in \mathbb{C}^{m\times n}$, $m \geq n$, rank $A = n$.

a) Show that the operation count for the modified Gram-Schmidt method is $2mn^2$ (the same as for the classical Gram-Schmidt method).

b) Let $m = n = 3$ and $A = QR$. In class we showed that the modified Gram-Schmidt method can be expressed in matrix form, $AR_1R_2R_3 = Q$, with $(R_1R_2R_3)^{-1} = R$, where $R_i, R$ are upper triangular matrices with positive diagonal elements. Show that the classical Gram-Schmidt method can also be expressed in matrix form.