1. On hw1 (problem 3.1, page 24) it was shown that if $||x||$ is a vector norm and $W$ is a nonsingular matrix, then $||x||_W = ||Wx||$ is a new vector norm. Let $||A||$ be the matrix norm induced by the original vector norm $||x||$ and let $||A||_W$ be the matrix norm induced by the new vector norm $||x||_W$. Show that $||A||_W = ||WAW^{-1}||$.

2. Let $A \in \mathbb{C}^{m \times n}$, $m \geq n$, rank $A = n$.
   a) Show that $A^* A$ is invertible.
   b) Let $P = A(A^* A)^{-1} A^*$. Show that $P^2 = P$, $P^* = P$, and range $P = \text{range} A$. (This shows that $P$ is the orthogonal projector onto range $A$.)
   c) Show that $P = \hat{Q} \hat{Q}^*$, where $A = \hat{Q} \hat{R}$ is the reduced QR factorization of $A$.

3. a) Let $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$. In class we applied the classical Gram-Schmidt method to compute the reduced QR factorization of $A$. Now apply Householder’s method to compute the full QR factorization of $A$. Work out the steps by hand.
   b) Using the results of part (a), find the orthogonal projector $P$ onto range $A$.

4. Let $A \in \mathbb{C}^{m \times n}$, $m \geq n$, rank $A = n$.
   a) Show that the operation count for mGS is $2mn^2$, the same as for cGS.
   b) Let $m = n = 3$ and $A = QR$. In class we showed that mGS can be expressed in matrix form, $AR_1 R_2 R_3 = Q$, with $(R_1 R_2 R_3)^{-1} = R$, where $R_i$, $R$ are upper triangular matrices with positive diagonal elements. Show that cGS can also be expressed in matrix form for some other choice of $R_1$, $R_2$, $R_3$.

**announcement** The midterm exam is in class on Thursday February 25. The exam will cover all the material up to and including the class on Tuesday February 23. You may use one page (one side) of notes. Calculators are not allowed.