MATH 571 Numerical Methods for Scientific Computing I Winter 2006

Assignment #7 due: Monday, April 24 (before 5pm, in my mailbox or office, not by email)

A membrane is described by a domain D in two space dimensions. The vibrations of the membrane are governed by the wave equation, $\phi_{tt} = \Delta\phi$, where $\phi(x, y, t)$ is the vertical displacement of the membrane. Assume that the edges of the membrane are clamped, so that we have a Dirichlet boundary condition, $\phi|_{\partial D} = 0$. Assume further that we're interested in a time-periodic vibration of the form $\phi(x, y, t) = e^{i\omega t}u(x, y)$, where ω is the vibration frequency. Then the function $u(x, y)$ satisfies the eigenvalue problem, $-\Delta u = \lambda u, u|_{\partial D} = 0$, where $\lambda = \omega^2$. Let D be the L-shaped domain shown in the figure. Find the smallest eigenvalue λ of the domain D. Present a contour plot and a surface plot of the corresponding eigenfunction. Solve the problem using the finite-difference approximation $\Delta u \approx D_+D_-u$ and any one of the e-value methods we discussed in class.