MATH 572 Numerical Methods for Scientific Computing II Winter 2005

hw#1 due: Tuesday, January 25

0. Write a brief description of your scientific interests and/or reasons for taking this course. If you work in a lab or research group, please give your supervisor's name and describe briefly the project you're working on. (1 paragraph is fine)

1. Consider the problem $y' = ay + b$, $y(0) = y_0$, where $a, b$ are constants and $a \neq 0$. Find the exact solution $y(t)$. Let $u_n$ be the numerical solution at time $t = nh$ given by Euler's method. Find an expression for $u_n$ and show that $\lim_{n\to\infty} u_n = y(t)$.

2. Consider the problem $y' = -y^2$, $y(0) = 1$.

a) Compute $y(1)$ using Euler's method with stepsize $h = 0.1, 0.05, 0.025, 0.0125$. Present the results in a table with the following information.

column 1: $h$
column 2: $u_n$
column 3: $u_n - y_n$
column 4: $(u_n - y_n)/h$

b) Find the principal error function $E(t)$ and evaluate $E(1)$. Compare with column 4.

c) Perform Richardson extrapolation on the values in column 2.

3. The numerical solution of the problem $y' = f(y)$, $y(0) = y_0$ by Euler's method has an asymptotic expansion of the form $u_n = y_n + hE_n + h^2 D_n + O(h^3)$ as $h \to 0$, where $E_n = E(t_n)$, $D_n = D(t_n)$ for certain functions $E(t)$, $D(t)$.

a) The equation for $E(t)$ was given in class. Find the equation for $D(t)$.

b) Consider the problem $y' = y$, $y_0 = 1$. In class we showed that $E(t) = -\frac{t}{2}e^t$. Find $D(t)$.

c) Compute $y(1)$ using Euler's method with stepsize $h = 0.1, 0.05, 0.025, 0.0125$. Present the results in a table with the following data.

column 1: $h$
column 2: $u_n$
column 3: $u_n - y_n$
column 4: $(u_n - y_n)/h$
column 5: $u_n - (y_n + hE_n)$
column 6: $(u_n - (y_n + hE_n))/h^2$

Evaluate $E(1), D(1)$ and compare with the results in the table.

4. Let $y' = f(y)$, where $f(y)$ is bounded and satisfies a Lipschitz condition. In class it was shown that an explicit 1-step method of the form $u_{n+1} = u_n + hF(u_n, h)$ converges if $F(u, h)$ satisfies two conditions: (a) $|F(u,h) - f(u)| \leq Ch$, (b) $|F(u,h) - F(v,h)| \leq \widetilde{L}|u - v|$, for $0 \leq h \leq h_0$, where $h_0$, $C$, $\widetilde{L}$ are positive constants. Find $F(u, h)$ for the midpoint rule and show that it satisfies these two conditions.