MATH 572 Numerical Methods for Scientific Computing II Winter 2005

Assignment #3 due : Tuesday, February 22

1. Solve the difference equation for u_n. Check and make sure your answer satisfies the initial conditions and difference equation.

a) $u_n - 2u_{n-1} - 3u_{n-2} = 0$, $u_0 = 0$, $u_1 = 1$

b) $u_n - 3u_{n-1} + 3u_{n-2} - u_{n-3} = 0$, $u_0 = 1$, $u_1 = 0$, $u_2 = -3$

2. Consider the following 2-step methods for $y' = f(y)$,

scheme 1 : $u_n - u_{n-2} - \frac{h}{3}(f(u_n) + 4f(u_{n-1}) + f(u_{n-2})) = 0$ (Milne's method)

scheme 2 : $u_n + 4u_{n-1} - 5u_{n-2} - h\big(4f(u_{n-1}) + 2f(u_{n-2})\big) = 0$

Answer the following questions for each scheme.

a) Find $\rho(\zeta)$, $\sigma(\zeta)$. Is the scheme consistent? Is the root condition satisfied?

b) Find the local truncation error in the form $\tau_n = Cy^{(r+1)}(t)h^{r+1} + O(h^{r+2})$.

c) Consider the equation $y' = \lambda y$. Find the characteristic roots and show that they satisfy

scheme 1 : $\zeta_1(h) = e^{h\lambda} + O(h^5)$, $\zeta_2(h) = -e^{-h\lambda/3} + O(h^3)$

scheme 2 : $\zeta_1(h) = e^{h\lambda} + O(h^4)$, $\zeta_2(h) = -5e^{-3h\lambda/5} + O(h^2)$

d) Take $y(0) = 1$ for the exact solution and $u_0 = 1$, $u_1 = e^{h\lambda}$ for the numerical solution. Derive the following expressions for u_n. Are the schemes convergent? Explain.

scheme 1 : $u_n = (1 + O(h^5)) \cdot \big(e^{\lambda t} + O(h^4)\big) + O(h^5) \cdot \big((-1)^n e^{-\lambda t/3} + O(h^2)\big)$

scheme 2 : $u_n = (1 + O(h^4)) \cdot \big(e^{\lambda t} + O(h^3)\big) + O(h^4) \cdot \big((-5)^n e^{-3\lambda t/5} + O(h)\big)$

3. Consider the van der Pol equation, $y'' + (y^2 - 1)y' + y = 0$, a model for nonlinear oscillations in an electrical circuit. The distinguishing feature of the equation is that it has a stable limit cycle, i.e. an attracting periodic solution. Express the equation as a first-order system, take $y(0) = 0.1$, $y'(0) = 0.1$ as initial data, and compute the numerical solution on the interval $0 \le t \le 40$ using the following schemes.

a) 2-step Adams-Bashforth

b) a predictor-corrector scheme using 2-step Adams-Bashforth as the predictor and one iteration of 1-step Adams-Moulton as the corrector

Take $h = 0.3$, 0.15, 0.075. Use the exact initial data and one step of Euler's method for the starting values of the numerical solution. Plot the numerical solution two ways: (1) in physical space (y versus t), (2) in the phase plane (y' versus y). Discuss the results. How do the two methods compare?

Announcement. The midterm exam is on Thursday, February 24, in class. It will cover all the material on ODEs. You may use one page of notes (one side) and a calculator.