

Assignment #7 due : Tuesday, April 19

LF = Lax-Friedrichs , LW = Lax-Wendroff

1. Consider the scalar 1D wave equation  $v_t + cv_x = 0$  with  $c > 0$ .

a) Plot (using Matlab) the amplification factor  $\rho(\xi h)$  in the complex plane for the upwind, LF, and LW schemes, for  $\lambda = 1/4, 1/2, 3/4, 5/4$ . Make a plot for each value of  $\lambda$ , showing all three methods on the same plot. Include a dotted line showing the unit circle. How are the methods similar? different?

b) Find the model equation for the LF and LW schemes. Compare these to the model equation for the upwind scheme (derived in class). Which scheme has the most artificial viscosity - upwind, LF, or LW? Which scheme has the least?

c) The numerical wave speed has an expansion of the form  $\tilde{c} = c(1 + \gamma_1 \xi h + \gamma_2 (\xi h)^2) + \dots$  in the long wave limit  $\xi h \rightarrow 0$ . Find  $\gamma_1, \gamma_2$  for the upwind and LF schemes (LW was done in class). Compare the size of the phase error in the three schemes.

2. Consider the LF scheme for  $v_t + cv_x = 0$ , where  $c$  may be positive or negative.

a) Show that the scheme is stable in the  $\infty$ -norm if  $|c|\lambda \leq 1$ .

b) Show that the scheme converges in the  $\infty$ -norm if  $|c|\lambda \leq 1$ .

3. Problem 3 on hw6 asked you to compute the solution of  $v_t + v_x = 0$  using the upwind scheme for two initial data. Repeat now using LF and LW. Are the present results better or worse than those obtained using the upwind scheme?

4. Consider the equation  $\phi_t + \phi_{xxx} = 0$  and the forward Euler/centered-difference scheme,  $u_j^{n+1} = u_j^n + kD_0D_+D_-u_j^n$ . What condition on the parameters  $h, k$  is necessary to ensure stability of the scheme in the 2-norm?

### Note

The final exam is on Wednesday, April 27, 4-6pm in room 130 Dennison. The exam will cover the entire course, but PDEs will be emphasized. You may use two pages of notes (i.e.  $2 \times 8.5 \text{ in} \times 11 \text{ in}$ ). Calculators are not allowed.