MATH 671 Particle Methods in Scientific Computing Winter 2007

hw#1 due: Tuesday, January 30

0. Give a brief description of your academic background and research interests. If you work in a lab or research group, please give your supervisor's name and describe your project. One paragraph is fine.

1. Compute the DFT of the following vectors.

a) $v = (1, 0, 0, 0)^T$ b) $v = (1, 1, 1, 1)^T$ c) $v = (1, 0, 1, 0)^T$ d) $v = (1, -i, -1, i)^T$

e) $v = (\sin 2\pi\omega t_j)^T$ where $j = 0 : N - 1$, $t_j = j/N$ and the frequency ω is not an integer

2. The Danielson-Lanczos lemma in the case $M = 2$ implies that $F_4 = \frac{1}{\sqrt{2}} B_4(F_2 \oplus F_2)P_4$. Write out each factor explicitly and check the equality.

3. Derive a Danielson-Lanczos lemma for the case $N = 3M$.

4. Express $n = 2007$ in binary form. Find the periodic shift n' and bit reversed index n''.

5. The discrete sine transform (DST) was defined in class for a vector $v \in \mathbb{C}^{N-1}$ by the formula $\widehat{v}_n = \sqrt{\frac{2}{N}} \sum_{j=1}^{N-1} v_j \sin\frac{\pi n j}{N}$ for $n = 1 : N - 1$. Find the formula for the inverse DST.

6. Plot the balanced and unbalanced trigonometric interpolants of the given function $v(x)$ on the interval $0 \le x \le 1$, as in class. Take $N = 4, 8, 16, 32$. In which cases does the interpolant converge uniformly to the given function?

a) $v(x) = 1 - 2|x - \frac{1}{2}|$ b) $v(x) = \sin \pi x$

7*. Write Matlab codes for the direct DFT and the FFT. The code should input a vector v and integer $N = 2^q, q \ge 1$ and output the vector $\widehat{v} = F_N v$. Consider the case $v = (1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{N})^T$ and compute $\widehat{v} = F_N v$ for $N = 2^q, q = 1 : 10$. Plot the required cpu time as a function of N using log-log scales for three codes: your direct DFT, your FFT, and Matlab's FFT. To demonstrate the correctness of your codes, present a list containing the vector component $\widehat{v}_{N-1}$ for all three methods, for the case $N = 10$ (which should agree up to roundoff). Discuss the results.

$$\Rightarrow \quad \lambda = \frac{-\omega^{-n} + (2 + \sigma^2 h^2) - \omega^n}{h^2} = \frac{2 + \sigma^2 h^2 - (e^{2\pi i n/N} + e^{-2\pi i n/N})}{h^2}$$

3. $C = F_N^* D F_N$, where $D = \mathrm{diag}(\sqrt{N}\widehat{c}_n)$

8*. Consider the BVP discussed in class, $-\phi'' + \sigma^2\phi = f$ on $0 \le x \le 1$ with periodic boundary conditions $\phi(0) = \phi(1)$, $\phi'(0) = \phi'(1)$. Let $\sigma = 2, f(x) = 1 - 2|x - \frac{1}{2}|$. Find the exact solution $\phi(x)$ by analytically evaluating the Green's function integral. Check that the result satisfies the differential equation and boundary conditions. Write codes to solve the problem numerically using the three methods discussed in class: (1) finite-difference scheme/FFT, (2) pseudospectral, (3) Green's function. Take $N = 4, 8, 16, 32$. Plot the exact solution and the numerical solution. Present a table with the following data. column 1: h, column 2: $\max|\phi_n - u_n|$, column 3: $\max|\phi_n - u_n|/h^{-2}$, column 4: $\mathrm{cond}_2(A)$. Discuss the results.