Math 555, Homework 7

Due Tuesday, November 8

Assignment:

1. [pp. 238–239] Chap 6, Sec. 67, Problem 2.
2. [pp.238–239] Chap 6, Sec. 67, Problem 3
3. [pp.245–246] Chap 6, Sec. 69, Problems 2(a), 3(a) and 3(b).
4. [pp.245–246] Chap 6, Sec. 69, Problem 5.
5. [pp.245–246] Chap 6, Sec. 69, Problems 7 and (8b).
6. [pp. 257–259] Chap 7, Sec. 72, Problem 3
7. [pp. 257–259] Chap 7, Sec. 72, Problem 6.
8. [pp. 257–259] Chap 7, Sec. 72, Problem 8.
9. Show that the following two statements are equivalent.

(a) The power series $\sum_{n=0}^{\infty} a_n z^n$ converges absolutely for all $|z| < R$.

(b) For any $R' < R$, there is a constant C depending on R' such that

$$|a_n| < C(\frac{1}{R'})^n, \ for\ all\ n \geq 0.$$

[You may consider $R' = R^{1-\epsilon}$ for some $\epsilon > 0$.]

10. Suppose that $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is analytic on $|z| < R_2$ and and $g(z) = \sum_{n=0}^{\infty} b_n (\frac{1}{z})^n$ is analytic on $|z| > R_1$, and that $R_1 < R_2$.

(a) Using theorems in text, show that $f(z)g(z)$ has a Laurent expansion $f(z)g(z) = \sum_{-\infty}^{\infty} c_n z^n$ around $z_0 = 0$ that is valid (converges absolutely) for any z in the annulus $R_1 < |z| < R_2$.

(b) Show that for each integer n that the sum

$$\sum_{k=0}^{\infty} a_k b_{k-n}$$

converges absolutely. (In this sum we set $b_{k-n} = 0$ if $k - n < 0$.) [Hint: Use result of problem 9 to bound $|a_k| < C_1(\frac{1}{R_2'})^k$, for any $R_2' < R_2$ and $|b_k| < C_2(R_1')^k$ for any fixed $R_1' > R_1$.]

(c) (*) Show that the Laurent series coefficients in (a) are $c_n = \sum_{k=0}^{\infty} a_k b_{k-n}$.