Math 555, Homework 8

Due Tuesday, November 15

Assignment:

1. [pp. 265-266] Chap. 7, Sec. 74, Prob. 3]

2. [pp. 265-266] (a) Chap. 7, Sec. 74, Prob. 9]

(b) Is the principal value needed on this problem? That is, is the integral have a value as an improper integral? What about the integral in Sec. 74, Prob. 10?

3. [pp. 276–278] Chap. 7, Sec. 77, Prob. 3.

4. [pp. 276–278] Chap. 7, Sec. 77, Prob. 6(b).

5. [pp. 280] Chap. 7, Sec. 78, Prob. 3.

6. [pp. 285–287] Chap. 7, Sec. 80, Prob. 1b, 1c.

7. [pp. 285–287] Chap. 7, Sec. 80, Problem 6.

8. (Ungraded) [pp. 296–297] Chap. 7, Sect 82, Problem 4.

9. (Ungraded) [pp. 265–266] Chap. 7, Sec. 74, Problem 12.

10 . (Ungraded) Define the improper integral

$$F(z) := \int_0^\infty e^{-t} t^{z-1} dt,$$

where we set $t^z := exp(zLogt)$, using the principal branch of the logarithm.

(a) Show that for $Re(z) > 0$ the integral for $F(z)$ converges absolutely. [Note: This integral defines an analytic function of z for $Re(z) > 0$ because the integral converges uniformly on the region $R_\epsilon := \{z : \epsilon \leq Re(z) \leq \frac{1}{\epsilon}\}$ for any positive ϵ.]

(b) Show that for $Re(z) > 0$ the function satisfies the identity $zF(z) = F(z+1)$. [Hint: Integrate by parts.]

(c) Show that the function $F(z)$ analytically continues to the whole complex plane except for simple poles at $z = 0, -1, -2, -3, ...$ [Hint: use the identity in (b). Define the function $G(z) := \frac{1}{z}F(z+1)$ for $Re(z) > -1$ by its right side. Then show $G(z) = F(z)$ for $Re(z) > 0$ by the identity in (b), hence this defines an analytic continuation to $Re(z) > -1$, except for a singularity at $z = 0$, which is a simple pole. (Why?) Now the right side of defn. of $G(z)$ is defined for $Re(z) > -2$ and this defines the left side for $Re(z) > -2$. Continue.]

(d) Evaluate $F(n)$ for n a positive integer.

Challenge. Can you evaluate $F(\frac{1}{2})$? [Hint: use information from Problem 9 statement.]