Coding Theory: Math 567
Take-Home Final Exam- Clarifications/Hints

1. Problem. The source variable X is a single bit. Let a variable U be the encoded version of X, so that $U \in \{000, 111\}$, with $X = 0$ encoded as 000, $X = 1$ as 111. The output variable Y is the result of transmitting U through the channel.

The variable Z is a single bit, giving the decoder's prediction of the value of X, computed by decoding Y.

(f) Compute the joint probability distribution (X, Z). ($X = 0$ was imposed to reduce the computations, but the whole distribution is needed for the other part of the question.)

(g) [Rewording] [10 pts] Determine the mutual information $I(X;W)$, in bits (i.e. source bits per transmitted channel symbol)? Then determine the amount of source information learned using the coding/decoding method in parts (a), (b), (f) above, measured in bits/transmitted channel symbol. Which method transmits more source bits/transmitted channel symbol? [A "transmitted symbol" refers to a transmission over the channel.]

[Bonus question] Here is an additional part to problem (1).

(i) [Bonus Problem-10 pt] Let the variable V be the error syndrome (coset leader) constructed by the decoding method from the received Y; it consists of three bits. Determine the joint probability distribution of (Z, V, Y). Then determine $H(V)$, $H(V, Z|Y)$, and $I(V; Z)$. (This may be time consuming.)

3. Problem. In parts (3b), (3c) describe what information that you can deduce about parameters (n', M', d') (Upper and lower bounds, or exact bounds, when possible.) [Hint: You might be able to use information on the nonexistence of codes of various sizes, i.e. known values of $A_2(n, d)$.]

In the bonus part (d) it might be useful to consider what the generator matrix of such a code would look like.

6. Problem.[Revised problem (a), (b)] ([20,12]-code to [21,12]-code)

Let C be a $[21, 12]$ double-error correcting binary linear code.

(a) [5 pts] How many coset leaders does this code have?

(b) [10 pts] What can you infer about how many coset leaders are there of each of the weights 1, 2, 3? (You may not necessarily get exact answers in all cases.) Justify your answers.

(c) [10 pts] Prove that such a code exists. [Hint: Consider constructing it from known codes by a "new code from old" method.]

7. Problem. In part (b) Table 4.1.5 gives information that the best code has either 36 or 37 words. The intent of part (b) is that you obtain the best bounds you can, not using this result.

In part (c) the asymptotic quantity $\alpha_q(\delta)$ is defined on page 192.