Math 567, Homework 1

Due Thursday, January 20

Assignment:

1. (a) Do §1.1 Exercise 3

(b) Do §1.1 Exercise 9

2. Do §1.1 Exercise 12

3. The world series is a seven game series that terminates as soon as a team wins four games. Let X be the random variable that represents the outcome of a world series between teams A and B; possible outcomes are AAAA, ABABABB, BBBAAAB. Let Y be the number of games played, which runs from 4 to 7. Assume teams A and B are evenly matched, and that the games are independent. Compute H(X), H(Y), and the conditional entropies $H(X|Y)$ and $H(Y|X)$.

4. Do §1.1 Exercise 18

5. Do §1.2 Exercise 7

6. Do §1.2 Exercise 9.
Note that if f is a random variable with range $\{x_1, x_2, \cdots x_n\}$ and
$f : \{x_1, x_2, \cdots x_n\} \to \{y_1, y_2, \cdots y_m\}$
is a map, then Y is a random variable with range $\{y_1, y_2, \cdots y_m\}$ such that
$Prob[Y = y_i] = \sum_{j:f(x_j)=y_i} Prob(X = x_j)$.

7. Do §1.2 Exercise 10. [Use convexity].

8. [Bonus problem.] A (wide sense, ergodic) stationary source $x_1, x_2, x_3...$ is one such that any pattern of $k \geq 1$ letters has a limiting frequency. That is, the quantities $Prob(x_i = S_1, x_{i+1} = S_2, ...s_{i+k-1} = S_k)$ is defined for any pattern $S_1 S_2 \cdots S_k$ of symbols, (and does not depend on i). Let H_k denote the entropy of the joint probability distribution $Prob(x_i = S_1, x_{i+1} = S_2, ...x_{i+k-1} = S_k)$ on k letters. Prove that the limit

$$H := \lim_{k\to\infty} \frac{1}{k} H_k$$

exists. (This limit is called the entropy of such a source.)
[Hint: Use Theorem 1.2.5].