

**Due Thursday, February 17**

**Assignment:**

Do your choice of 7 of the following 10 problems. (Do an eighth problem as bonus problem, bonus problem must be either prob. 9 or 10, please work part 9(c) for bonus if doing prob. 9 for bonus)

1. (a) Do Sect. 3.1 problem 5.  
(b) Do Sect. 3.2 problem 3.
2. Do Sect. 3.1 Problem 16.
3. Do Sect. 3.2 problem 5.
4. Do Sect. 3.3 problem 2.
5. Do Sect. 3.3 problem 6.
6. Do Sect. 3.3 problem 7.
7. Do Sect. 3.3 problem 10.
8. Do Sect. 3.4 problem 4.

9. Suppose you have 12 coins, such that exactly one of the coins is counterfeit and is lighter than the other eleven coins (which all have the same weight). All coins are equally likely to be counterfeit. You may attempt to detect the counterfeit coin making exactly two weighings on a scale, comparing weights of two subsets of the coins of your choice. The outcome is either they weigh the same or one side is heavier than other (3 possible outcomes).

(a) Show using entropy (or otherwise) that there is no scheme to determine the counterfeit coin with complete certainty using two weighings. [Let  $X$  be distribution of location of counterfeit coin and  $Y$  output of weighings. What can you say about  $H(X|Y)$ ?]

(b) Using Fano's inequality, determine some positive lower bound for the decision error probability valid for any possible weighing procedure.

(c) [Optional] Find and compute the best upper bound you can for the (average) probability of decision error, describing your decision procedure (weighing sequence) to do it.

10. (Nonsymmetric channel) Determine the capacity of the nonsymmetric binary channel, where input 0 has output 0 with probability 1 and where input 1 has output 0 or 1 with probability  $1/2$  each. Also give the input probability distribution that achieves the capacity.