

Math 575, Homework 2 Bonus (due Monday, Sept. 24).

6. (*) (a) Find all solutions (a_1, a_2, a_3) with $a_i > 1$ such that

$$a_1 a_2 \equiv 1 \pmod{a_3}$$

$$a_2 a_3 \equiv 1 \pmod{a_1}$$

$$a_3 a_1 \equiv 1 \pmod{a_2}.$$

(Hint: Prove $\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} > 1$.)

(b) Show for each $r \geq 3$ there are only finitely many (a_1, \dots, a_r) with each $a_i > 1$ that satisfy

$$\frac{M}{a_i} \equiv 1 \pmod{a_i}, \quad \text{for } 1 \leq i \leq r,$$

where $M := \prod_i a_i$.

(More bonus: Can you get an upper bound for how many such (a_1, \dots, a_r) there are?)