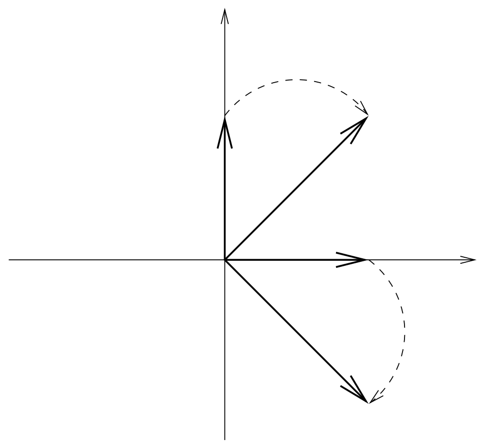
Problem # 2. Let us see where do the basis vectors go when we apply the transformation:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$



Hence the transformation is the rotation through an angle of 45° clockwise followed by the dilation by a factor of $\sqrt{2}$. From this, we deduce that the transformation is invertible and that the inverse transformation is the rotation through an angle of 45° counterclockwise followed by the scaling by a factor of $1/\sqrt{2}$. We compute the inverse matrix

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & 1/2 & 1/2 \end{array}\right].$$

Answer: a) The transformation is the rotation through an angle of 45° clockwise followed by the dilation by a factor of $\sqrt{2}$. b) The transformation is invertible, the inverse transformation is the rotation through an angle of 45° counterclockwise followed by the scaling by a factor of $1/\sqrt{2}$. The matrix if the inverse transformation is $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$.