Problem # 1. Reducing the matrix, we get

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 3 & 3 & 1 \\ 3 & 5 & 4 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -2 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The kernel of A consists of the solutions of the system $A\vec{x} = \vec{0}$, so the kernel of A consists of the vectors

$$\begin{bmatrix} -3x_3 - 2x_4 \\ x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

where x_3 and x_4 can be any numbers. From that, vectors $\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ constitute a basis of the kernel and the kernel is a plane in $\mathbf{R}^4$.

To choose a basis of the image of A, we choose the columns of A corresponding to the leading 1's of the reduced row-echelon form, that is, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, from which the image is a plane in $\mathbf{R}^3$.

Answer: The kernel is a plane in $\mathbf{R}^4$ with a basis $\begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ and the image is a plane in $\mathbf{R}^3$ with a basis $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$.