2. Find all values of the parameters $a$ and $b$ for which the system

\[ \begin{align*}
  x + ay &= 1 \\
  2x + 3y &= b
\end{align*} \]

a) has no solutions  
b) has a unique solution  
c) has infinitely many solutions.

**Solution.** Writing the system in the matrix form and reducing it, we get

\[
\begin{bmatrix}
  1 & a & 1 \\
  2 & 3 & b
\end{bmatrix} \rightarrow \begin{bmatrix}
  1 & a & 1 \\
  0 & 3 - 2a & b - 2
\end{bmatrix}.
\]

Now, if $a \neq 3/2$ then $3 - 2a \neq 0$ and we can find $y$ from the second equation as $y = (b - 2)/(3 - 2a)$ and then find $x$ from the first equation as $x = 1 - ay$. Hence in this case the system has a unique solution.

If $a = 3/2$ and $b \neq 2$ then the system has no solutions since the last equation reads $0 \cdot x + 0 \cdot y = b - 2 \neq 0$.

If $a = 3/2$ and $b = 2$ then the last equation reads $0 = 0$ and the system has infinitely many solutions since we can let $y$ be any number and then find $x = 1 - ay$.

**Answer.**  
If $a = 3/2$ and $b \neq 2$, the system has no solutions.  
If $a \neq 3/2$, the system has a unique solution.  
If $a = 3/2$ and $b = 2$, the system has infinitely many solutions.