2. Compute the orthogonal projection of the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$ onto the plane spanned by $\begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$.

Solution. We look for the orthogonal projection in the form $x\begin{bmatrix}1\\1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\\1\end{bmatrix}$ such that the difference $\begin{bmatrix}1\\1\\1\end{bmatrix} - x\begin{bmatrix}1\\1\\0\end{bmatrix} - y\begin{bmatrix}0\\1\\1\end{bmatrix}$ is perpendicular to both $\begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$. This gives us the equations

$$\left(\begin{bmatrix}1\\1\\1\end{bmatrix} - x\begin{bmatrix}1\\1\\0\end{bmatrix} - y\begin{bmatrix}0\\1\\1\end{bmatrix}\right)\cdot\begin{bmatrix}1\\1\\0\end{bmatrix} = 0 \quad \text{and}$$
$$\left(\begin{bmatrix}1\\1\\1\end{bmatrix} - x\begin{bmatrix}1\\1\\0\end{bmatrix} - y\begin{bmatrix}0\\1\\1\end{bmatrix}\right)\cdot\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

that is,

$$2 - 2x - y = 0 \quad \text{and}$$
$$2 - x - 2y = 0.$$

Solving the system (computations omitted), we get $x = y = 2/3$, so the orthogonal projection is $\frac{2}{3}\begin{bmatrix}1\\1\\0\end{bmatrix} + \frac{2}{3}\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}2/3\\4/3\\2/3\end{bmatrix}$.

Answer. The orthogonal projection is $\begin{bmatrix}2/3\\4/3\\2/3\end{bmatrix}$.