

1. Suppose that $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigenvector of a matrix A and that the corresponding eigenvalue is 2. Let $B = A^2 - 3A$. Compute $B\vec{v}$.

Solution. We have $A\vec{v} = 2\vec{v}$. Therefore,

$$\begin{aligned} B\vec{v} &= (A^2 - 3A)\vec{v} = A^2\vec{v} - 3A\vec{v} = A(A\vec{v}) - 3(2\vec{v}) = A(2\vec{v}) - 6\vec{v} \\ &= 2A\vec{v} - 6\vec{v} = 2(2\vec{v}) - 6\vec{v} = 4\vec{v} - 6\vec{v} = -2\vec{v} = \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}. \end{aligned}$$

Answer. $B\vec{v} = \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$.