Math/Stat 425, Solutions to Quiz #8

Problem # 2. Let X and Y be jointly distributed as

$$f_{X,Y}(x,y) = 2(x+y) \ \text{ for } 0 \le x \le y \le 1.$$

and $f(x,y) = 0$ otherwise. Determine the conditional density function $f_{X|Y}(x|y)$, for $0 < y \le 1$ (for all values of x).

Solution: The support of the density is the triangular region in the upper left of the unit square, above the line $x = y$. (This is the condition $x < y$.) By definition

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}, \qquad -\infty < x < \infty,$$

provided that $f_Y(y) \ne 0$. We must determine the marginal density

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$$

For $0 < y \le 1$ we have

$$\begin{aligned} F_Y(y) &= \int_0^y 2(x+y)dx \\ &= [x^2 + 2xy|_{x=0}^{x=y} = y^2 + 2y^2 = 3y^2. \end{aligned}$$

We obtain, for $0 < y \le 1$,

$$f_{X|Y}(x|y) = \frac{2(x+y)}{3y^2} \ \text{ for } \ 0 \le x \le y.$$

By definition $f_{X|Y}(x|y) = 0$ otherwise, i.e. if either $x < 0$ or $x > y$. (The density $f_{X,Y}(x,y) = 0$ at these points, and $f_Y(y) \ne 0$.

Answer. For $0 < y \le 1$,

$$\begin{aligned} f_{X|Y}(x|y) &= \frac{2(x+y)}{3y^2} \ \text{ for } \ 0 \le x \le y, \\ f_{X|Y}(x|y) &= 0 \ \text{ for } \ x < 0 \ \text{ or } \ x > y. \end{aligned}$$