A brief overview of the work of Shing-Tung Yau

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Abstract

The purpose of this article is to give a brief overview of the work of Yau by compiling lists of his papers and books together with brief comments on some part of his work, and describing a sample of some open problems raised by him. It tries to complement other more detailed commentaries on particular aspects of his work by experts in this volume. We hope that such an overview will convey both the depth and width of his work.

Contents

1 Introduction

Shing-Tung Yau has revolutionized the broad field of geometric analysis by combining partial differential equations with differential geometry and applying geometric analysis to algebraic geometry. Besides his celebrated solution to the Calabi conjecture on Kähler-Einstein metrics and applications of the Calabi-Yau manifolds to the string theory and the mirror symmetry, he has also initiated highly original approaches to harmonic analysis on manifolds by using gradient estimates and Harnack inequalities, topology of manifolds using minimal surfaces, rigidity of complex manifolds and algebraic varieties using harmonic maps, and the positive mass conjecture in general relativity using minimal surfaces. More recently, he also initiated applications of sophisticated mathematical methods to computer imaging and other applied mathematics.

The basic purpose of differential geometry and relations between differential geometry and differential equations are explained by Yau in [86-8]:

"*The basic purpose of geometry is to give a good description of a class of geometric objects. Usually this means that we have to give a good description of analytic structures over a space and the geometric objects defined by such structures. In many cases, we have to know how to deform these structures and study the dynamics of the geometric objects within these structures. The description of all these geometric phenomena usually are governed by differential equations. As geometric objects are in general curved, most of these equations are nonlinear.*"

The role of nonlinear differential equations in the study of geometry is emphasized by Yau in [80-7]:

"*One of the main purposes of differential geometry is to understand how a surface (or a generalization of it) is curved, either intrinsically or extrinsically. Naturally, the problems that are involved in studying such an object cannot be linear. Since curvature is defined by differentiating certain quantities, the equations that arise are nonlinear differential equations.*"

Indeed, nonlinear differential equations have played an important role in the work of Yau. Because of Yau's enormous contributions to many fields, he has received many distinguished awards. As mentioned in the citation of the Veblen Prize for Yau in 1981,

"*We have rarely had the opportunity to witness the spectacle of the work of one mathematician affecting, in a short span of years, the direction of whole areas of research.... Few mathematicians can match Yau's achievements in depth, in impact, and in the diversity of methods and applications.*"

In 1983, Yau was awarded a Fields medal. In describing Yau's work for occasion, L.Nirenberg wrote:

"*Yau has done extremely deep work in global geometry and elliptic partial differential equations, including applications in three-dimensional topology and in general relativity theory. He is an analyst's geometer (or geometer's analyst) with remarkable technical power and insight. He has succeeded in solving problems on which progress had been stopped for years.*"

More than ten years later, Yau was awarded the Carfoord prize in 1994, and the citation of the award says:

"The Prize is awarded to … Shing-Tung Yau, Harvard University, Cambridge, MA, USA, for his development of non-linear techniques in differential geometry leading to the solution of several outstanding problems.

Thanks to Shing-Tung Yau's work over the past twenty years, the role and understanding of the basic partial differential equations in geometry has changed and expanded enormously within the field of mathematics. His work has had an impact on areas of mathematics and physics as diverse as topology, algebraic geometry, representation theory, and general relativity as well as differential geometry and partial differential equations. Yau is a student of legendary Chinese mathematician Shiing-Shen Chern, for whom he studied at Berkeley. As a teacher he is very generous with his ideas and he has had many students and also collaborated with many mathematicians."

Most recently, Yau received the Wolf Prize in 2010, and the citation says:

"*Shing-Tung Yau (born 1949, China) has linked partial differential equations, geometry, and mathematical physics in a fundamentally new way, decisively shaping the field of geometric analysis. He has developed new analytical tools to solve several difficult nonlinear partial differential equations, particularly those of the Monge-Ampere type, critical to progress in Riemannian, Kähler and algebraic geometry and in algebraic topology, that radically transformed these fields. The Calabi-Yau manifolds, as these are known, a particular class of Kähler manifolds, have become a cornerstone of string theory aimed at understanding how the action of physical forces in a high-dimensional space might ultimately lead to our four-dimensional world of space and time. Prof. Yau's work on T-duality is an important ingredient for mirror symmetry, a fundamental problem at the interface of string theory and algebraic and symplectic geometry. While settling the positive mass and energy conjectures in general relativity, he also created powerful analytical tools, which have broad applications in the investigation of the global geometry of space-time.*

Prof. Yau's eigenvalue and heat kernel estimates on Riemannian manifolds, count among the most profound achievements of analysis on manifolds . He studied minimal surfaces, solving several classical problems, and then used his results, to create a novel approach to geometric topology. Prof. Yau has been exceptionally productive over several decades, with results radiating onto many areas of pure and applied mathematics and theoretical physics."

The results and their impacts mentioned at the beginning of this introduction and in the citations above are only a fraction of his work up to today, and probably many people are only aware of small parts of his work.

In this volume, we have asked many experts from different areas to comment on various aspects of his work. Their titles and authors are as follows:

1. Wen-Lin Chiou, Jie Huang, Lizhen Ji, *The work of Yau on filtering problem*,
2. Fan Chung, *From continues to discrete: Yau's work on graph theory*,
3. Chuck Doran, *Yau's Work on Moduli, Periods, and Mirror Maps for Calabi-Yau Manifolds*,
4. Felix Finster, *Review on Shing-Tung Yau's work on the coupled Einstein equations and the wave dynamics in the Kerr geometry*,
5. Gregory J. Galloway, *The work of Witten and Yau on connectedness of the boundary in the AdS/CFT correspondence*,
6. Alexander Grigor'yan, *Yau's work on heat kernels*,

7. Xianfeng Gu, *Professor Yau Contributions to Engineering Fields*,
8. Conan Leung, *The SYZ proposal*,
9. Conan Leung, *Yau-Zaslow formula*,
10. Peter Li, *Yau's work on function theory: harmonic functions, eigenvalues and the heat equation*,
11. Bong Lian, *A Vision of Shing-Tung Yau on Mirror Symmetry*,
12. Kefeng Liu, *Yau's works on group actions*,
13. John Loftin, Xu-Jia Wang and Deane Yang, *Cheng and Yau's work on the Monge-Ampère equation and affine geometry*,
14. Feng Luo, *Yau's work on minimal surfaces and 3-manifolds*,
15. William Minicozzi, *The work of Schoen and Yau on manifolds with positive scalar curvature*,
16. Andrey Todorov, *Professor Yau Contributions to Algebraic Geometry*,
17. Mu-Tao Wang, *Professor Shing-Tung Yau's work on positive mass theorems*,
18. Xiaowei Wang, *Yau's conjecture on Käahler-Einstein metric and stability*,
19. Fangyang Zheng, *On Yau's Pioneer Contribution on the Frankel Conjecture and Related Questions*,
20. Kang Zuo, *Yau's work on inequalities between Chern numbers and uniformization of complex manifolds.*

Besides describing his work in many different areas, these articles also try to explain the history of the problems before the work of Yau, and further developments after his work.

On the other hand, the purpose of the current article is very different and tries to give a very brief overview of his contributions by making lists of various things such as topics he has worked on, papers and books written by him, and students trained by him etc. Even with this modest purpose, it is still beyond the ability of the author to present a glimpse of his work and hence to give a sense of their scale and to do justice to his enormous output. Our only hope is that such a brief overview might help convey a sense of his global approach to mathematics and complement the above papers written by the experts on specific topics.

In spite of the above list of many articles by experts on Yau's work, they do not cover many topics of his work, for example, his results on harmonic maps and applications to rigidity of locally symmetric spaces and Kähler manifolds. In view of this, we have added, with the generous help from J. Jost, some more detailed comments on this aspect of his work in this article in §6, §7 and §8. It should be clear to some readers that many other aspects of his work are still left out, for example, his work on compactifications of negatively curved Kähler manifolds of finite volume, and his extensive study on metrics of the moduli spaces of Riemann surfaces.

It should be also pointed out that many of Yau's preprints and ongoing projects are not listed or mentioned in this article. The first part of the references consists of almost all published papers of Yau listed according to the year of publication in MathSciNet[1], and the second part consists of some papers by others which are cited in the article or related to the discussion here.

Since educating students is another major contribution of Yau towards math, we have also added near the end the list of all his Ph.D. students up to now the summer of 2009.

Acknowledgements. I would like to thank Juergen Jost for his generous contribution to the comments on Yau's work related to harmonic maps, rigidity of Kähler manifolds and locally symmetric spaces in §6, §7 and §8.

2 A summary of some major works of Yau

In this article, we have tried to classify all papers of Yau into more than 80 topics. For the convenience of the reader, we highlight some of his achievements in this section. Whenever available, we had added references to one of the commentaries of Yau's work in this volume right after each result.

Major results proved.

1. A right definition of quasilocal mass in general relativity, one of the major unsolved problems in general relativity proposed by Penrose in 1982 (2009) [Wa].

2. Coupled Einstein equations and wave dynamics in the Kerr space-time geometry (2002-2009) [Fin].

3. Metrics on the moduli space of curves (2004-2005).

4. The complete solution of the mirror principle (1997-2002) [Lia].

5. SYZ conjecture on mirror symmetry (1996) [Leu1].

6. Yau-Zaslow conjecture (1996) [Leu2].

7. Connectedness of the boundary in the AdS/CFT correspondence (1999) [Gal].

8. Stability of vector bundles and Hermitian-Einstein metrics (1986) [Wan].

9. Harnack inequality and sharp bounds on the heat kernel (1986) [Li].

10. Rigidity of manifolds and harmonic maps (1985-1999) (see §6, §7 and §8).

11. Solution to the Smith conjecture and minimal surfaces (1981-1984) [Luo].

12. Solution to the Frankel conjecture on characterization of the complex projective space (1980) [Zhe].

13. Solution to the positive mass conjecture in general relativity (1979-1981) [Wa].

14. Classification of manifolds with positive scalar curvature and group actions (1979) [Min] [Liu].

[1] It seems that some papers of Yau are not listed in MathSciNet and we have tried to list those known to us.

15. Schwarz Lemma for complex manifolds (1978).
16. Solution of the Calabi conjecture on Kähler-Einstein metric (1977) [Tod2] [Wan].
17. Solution of the Severi conjecture on the homotopy rigidity of $\mathbb{C}P^2$ (1977) [Tod2].
18. Miyaoka-Yau inequality for Chern classes (1977) [Zuo].
19. Solution to the regularity of the Minkowski problem of prescribing Gauss curvature (1976) [Wang] [Lof].

Important methods initiated.

Yau has also initiated several important methods:

1. The method of gradient estimates for bounding harmonic functions and heat kernel [Li] [Wang].
2. The method of using minimal surfaces for the topology of manifolds, the positive mass in relativity theory, uniformization of complex manifolds [Luo].
3. The method of using harmonic maps for rigidity of locally symmetric spaces and Kähler manifolds with negative curvature (see §6, §7 and §8).
4. Adapt methods for smooth manifolds to discrete spaces and apply methods of differential and algebraic geometry to applied mathematics such as computer imaging [Gu] [Chu].

Profound open problems raised.

Yau has raised many important questions and made many conjectures. Several of them have lead to a lot of development and opened up new fields:

1. Uniformization of complex manifolds.
2. Sizes on the space of harmonic functions with polynomial growth.
3. Rank rigidity and geometry of nonpositively curved compact or finite volume manifolds.
4. Relations between Kähler-Einstein metrics on complex manifolds with positive first Chern class and stability of manifolds in the senses of algebraic geometry.

3 Topics Yau has worked on

Yau's work and interests are very broad. The following is a list of topics he has worked on roughly according to the dates of publications of the papers in MathSciNet with a major exception that more applied topics are listed in the second half.

For each topic, we have listed some related papers. Due to the lack of knowledge of the author, this list is almost surely not complete and probably does not reflect topics of the papers correctly.

1. Kähler-Einstein metrics with nonpositive first Chern class [77-1] [78-1].

2. Kähler-Einstein metrics with positive first Chern class [87-6] [09-4].
3. Noncompact Kähler-Einstein manifolds [91-4] [90-1] [90-3] [87-8] [87-9] [84-6] [83-2] [80-3].
4. Explicit constructions of Calabi-Yau manifolds [98-11] [91-7] [87-2] [87-9] [85-1] .
5. Miyaoka-Yau inequality between between Chern numbers [86-7] [87-8] [07-4].
6. Calabi-Yau equation [08-14].
7. Geometric approach to birational problems [08-10].
8. Geometry of the G_2 moduli space [09-2].
9. Sasaki-Einstein metrics [07-2] [06-5].
10. Monge-Ampére-type equation for non-Kähler manifolds and string theory [08-21] [07-10] [05-1].
11. Heterotic string theory [08-16].
12. Real Monge-Ampére equations [82-4] [78-1] [80-3] [77-3].
13. Projectively flat Hermitian manifolds [94-1] [90-4].
14. Compactifications in string theory [06-6] [90-1].
15. Mirror symmetry and SYZ program [05-12] [05-2] [04-7] [02-13] [01-3] [98-11] [96-1] [96-1] (or [01-2]).
16. Mirror principle, which states roughly that certain sequences of multiplicative equivariant characteristic classes on stable map moduli spaces can be computed in terms of certain hypergeometric type classes [06-9] [03-15] [03-18] [02-1] [02-6] [00-1] [99-1] [99-10] [99-4]) [97-2].
17. Local mirror symmetry, which is an application of techniques from mirror symmetry to Fano surfaces within Calabi-Yau manifolds [99-16].
18. Mirror map [03-10] [99-3] [98-2] [98-3] [97-15] [96-2] [96-5] [96-13] [95-1] [95-9] [95-10] [92-3].
19. Yau-Zaslow conjecture: enumerative geometry of K3 [96-3].
20. Duality and fibrations on G2 manifolds [03-17].
21. Fourier-Mukai transform [01-3] [01-7].
22. Moduli stacks [04-6].
23. Fourier-Mukai partners of K3 surfaces and Kummer structures of K3 surfaces [04-4] [03-9] [03-16].
24. Conformal field theory [03-12] [93-2].
25. Local heterotic torsional models [09-6].

26. Toric varieties [02-3].
27. Gromov-Witten invariants [04-5] [99-12] [97-2].
28. Metrics on moduli space of Riemann surfaces [08-20] [08-19] [08-17] [08-4] [05-10] [05-9] [05-6] [04-2] [80-7].
29. Moduli spaces of Calabi-Yau manifolds [05-11] [05-12] .
30. Positive mass conjectures and quasi-local masses in the general relativity theory [09-5] [09-1] [07-5] [06-3] [04-3] [03-13] [02-6] [96-6] [88-4] [82-10] [81-5] [81-2] [80-2] [79-3] [79-7] [79-6] [78-3].
31. Existence of black holes and geometry of three manifolds [01-1] [97-4] [83-5].
32. Solutions to the Einstein/Yang-Mills equation [95-5] [93-9] [92-10] [91-6].
33. Solutions to the Einstei-Dirac-Maxwell equation. [01-8] [00-7] [00-9] [00-10] [00-11] [99-7] [99-8] [99-9] [99-11].
34. Kerr-Newman geometry and Kerr geometry [06-8] [09-8] [05-5] [03-11] [02-14].
35. Connectedness of the boundary in the AdS/CFT correspondence [02-9] [02-10] [99-14].
36. Bounds on eigenvalues of Riemannian manifolds and isoperimetric inequalities [04-1] [03-5] [03-14] [00-5] [97-5] [96-4] [94-4] [85-2] [84-1] [83-1] [82-3] [81-7] [80-4] [80-5] [75-4].
37. Gap of eigenvalues of Schrodinger operators [08-1].
38. Effective inverse spectral geometry [92-8].
39. Distribution of critical points of eigenfunctions [97-1].
40. Harnack inequalities [95-2] [94-2] [92-5] [97-3].
41. Harmonic maps [94-6] [93-1] [91-1] [91-3] [79-4] [79-8] [78-5] [76-1].
42. Rigidity of spaces of Kähler manifolds [94-3] [93-3] [93-4] [93-14] [91-9] [87-11] [86-5] [85-4] [83-3] [76-6].
43. Super-rigidity of spaces of nonpositive curvature [99-6] [97-6] [93-5].
44. Uniformization of complex manifolds [96-12] [93-7] [93-13] [91-2] [91-8] [90-2] [88-1] [81-1] [77-4] [76-5].
45. Kähler manifolds of positive bisectional curvature: Frankel conjecture [80-6].
46. Schwarz lemma in higher dimensions [78-4].
47. Classification of algebraic surfaces [90-4] [90-5].
48. Classification of Kähler surfaces [74-1].

49. Geometry and topology of minimal surfaces [92-6] [82-5] [82-2] [79-1] [78-3].

50. Minimal hypersurfaces [75-3].

51. Nonlinear Schrödinger equations [88-2].

52. Manifolds with positive scalar curvature [87-1] [85-3] [79-5] [78-3].

53. Conformally flat manifolds[88-3].

54. Hermitian-Yang-Mills connections [89-1] [87-3] [86-4].

55. Complete affine hypersurfaces [86-2].

56. Heat kernel bounds [86-3] [84-2] [81-3] [81-6] [78-2].

57. Existence of negatively Ricci curved metrics on three-manifolds [86-6].

58. Smith conjecture and three dimensional topology [84-3] [84-4] [83-4] [82-6] [81-4] [82-1] [80-1] [79-2].

59. Group actions and topology [79-4] [77-2] [74-3].

60. Compactifications of Kähler manifolds of finite volume [82-8].

61. HyperKäahler manifolds and birational transformations [02-12].

62. Complex manifolds and birational geometry [75-6].

63. Hypersurfaces with constant scalar curvature [77-5].

64. Fundamental group and curvature the Lorentz-Minkowski spaces [72] [71-2] [70].

65. Bernstein problem [76-2].

66. Minkowski problem [76-3].

67. Liouville theorem and gradient estimate [76-4] [75-2] [75-5].

68. Submanifolds with constant mean curvature [75-1] [74-2].

69. Curvature preserving diffeomorphisms [74-4].

70. Convex function and infinite volume of manifolds [74-5].

71. non-immersibility in $\mathbb{R}^3$ of non-compact surfaces [73-1].

72. Conformal transformations [73-2].

73. Compact flat Riemannian manifolds [71-1].

74. Computer vision and computer graphics [05-8] [03-1] [03-3] [02-2].

75. Surface segmentation [05-7] [08-2].

76. Geometric compression using Riemann surface structure [03-2].

77. Discrete surface approximation [07-8].

78. Conformal parametrization [08-3] [08-5]]08-6] [07-2].

79. Geometric methods in engineering applications [08-7] [08-15].

80. Minimization with the affine normal direction [05-3].

81. Numerical analysis [91-5].

82. Filtering problem [05-4] [01-5] [00-6] [98-1] [98-5] [96-11].

83. Analysis on finite graphs [01-4] [00-14] [00-15] [99-21] [99-22] [97-10] [97-11] [97-12] [96-8] [96-10] [96-14] [95-7] [95-8] [94-7].

84. Elastic impact theory [00-4].

85. Matrix harmonics on S^2 [98-6].

86. Zero energy wave functions in supersymmetric matrix models [00-18].

87. von Kármán equations [92-7].

4 Basics on Kähler-Einstein metrics and Calabi conjectures

It is known that any smooth manifold M admits a Riemannian metric. A natural and basic question is whether there is any *best* or *canonical* Riemannian metric on it.

When M is an orientable surface, by the uniformization theorem, M always admits a Riemannian metric of constant curvature. If M is given a complex structure, then there is also a metric of constant curvature which is compatible with the complex structure, i.e., conformal to the complex structure.

A direct generalization of the uniformization theorem does not hold in higher dimension. The condition that the existence of a Riemannian metric of constant sectional curvature is too restrictive. It turns out that a natural choice of a good metric is the so-called *Einstein metric*, i.e., the Ricci curvature is constant. This condition was analogous to Einstein's field equation in general relativity, and hence the name of Einstein metric is used.

On the other hand, it is very difficult to construct Einstein manifolds which are not homogeneous spaces. The solution of the Calabi conjecture by Yau in [78-1] provides a large class of Kähler-Einstein manifolds. See the discussions in [Bes] about why this condition is natural. See also [LeB] for a more recent survey about Einstein manifolds.

More specifically, let M be a compact complex manifold with a Kähler metric $g = \Sigma_{i,j} g_{i\bar{j}} dz^i \otimes d\bar{z}^j$. Let $\frac{\sqrt{-1}}{2}\Sigma_{i,j} g_{i\bar{j}} dz^i \wedge d\bar{z}^j$ be its Kähler form. Let $\Sigma_{i,j} R_{i\bar{j}} dz^i \otimes d\bar{z}^j$ be its Ricci tensor, and $\frac{\sqrt{-1}}{2}\Sigma_{i,j} R_{i\bar{j}} dz^i \wedge d\bar{z}^j$ its Ricci form. The metric g is called a *Kähler-Einstein metric* if the Ricci form is proportional to the Kähler form. Around 1954 [Ca1, Proposition on p. 80 and the footnote on p. 88] (see also [Ca2]), Calabi conjectured existence of Kähler-Einstein metrics on Kähler manifolds M whose Chern class $c_1(M)$ is negative, zero, or positive with the additional condition that M does not admit any nontrivial holomorphic vector field. Note that the assumption that the first Chern class $c_1(M)$ has a sign is a necessary condition.

These conjectures of Calabi follow from slightly more general ones he formulated. Specifically, it is known that the Ricci tensor is given by $R_{i\bar{j}} = -\frac{\partial^2}{\partial z^i \partial \bar{z}^j}[\log \det(g_{i\bar{j}})]$, and hence the Ricci form $\frac{\sqrt{-1}}{2\pi}\Sigma_{i,j} R_{i\bar{j}} dz^i \wedge d\bar{z}^j$ is closed. It is also known that the cohomology class of the Ricci form represents the first Chern class $c_1(M)$. Calabi conjectured (or rather claimed in [Ca2]):

1. if a closed form $\tilde{\omega} = \frac{\sqrt{-1}}{2\pi}\Sigma_{i,j}\tilde{R}_{i\bar{j}} dz^i \wedge d\bar{z}^j$ represents $c_1(M)$, then there there exists a unique Kähler metric $\tilde{g} = \Sigma_{i,j} g_{i\bar{j}} dz^i \otimes d\bar{z}^j$ with $\tilde{\omega}$ as its Ricci form and the Kähler forms of g and $\tilde{g}$ belonging to the same cohomology class;

2. in particular, if $c_1(M) = 0$, we can take $\tilde{\omega} = 0$ and get as a corollary the existence of a *Ricci-flat Kähler-Einstein* metric.

In [78-1], Yau solved a stronger form of this conjecture and proved the existence of Kähler-Einstein metrics when the first Chern class is zero or negative. In the latter case, the metric is unique. Thrse results together with some applications were announced in [77-1].

5 Some applications of Kähler-Einstein metrics and Calabi-Yau manifolds

If a complex manifold admits a canonical Kähler-Einstein metric, the metric can be used to understand the geometry and topology of the manifold in a very effective way. The results obtained so far seem to indicate that this has mainly been the case when the Ricci curvature (or the first Chern class) is nonpositive.

For the convenience of the reader, we collect some major applications of the Calabi-Yau manifolds into the following list. Several were obtained by Yau in the original paper [77-1] where the solution of the Calabi conjecture was announced by him. It is conceivable that there should be more applications given that such a natural Kähler-Einstein metric greatly enhances the underlying manifold.

1. There exist compact simply-connected Kähler manifolds whose Ricci curvature is identically zero. Examples are given by a complex hypersurface in $\mathbb{C}P^n$ of degree $n+2$. Note that complex torus $\Lambda\backslash\mathbb{C}^n$ are flat Kähler manifolds but clearly not simply connected. The existence of the above simply connected manifolds is not obvious at all.

2. There exist compact-simply connected Kähler manifolds whose Ricci curvature is equal to a negative constant.

3. If M^n is a Kähler manifold withthe first Chern class $c_1(M)$ negative or zero, then
$$(-1)^n c_1^{n-2}(M)c_2(M) \geq (-1)^n \frac{n}{2(n+1)} c_1^n(M).$$
This is the celebrated Yau inequality. When M is a Kähler surface with ample canonical bundle, it specializes to
$$3c_2(M) \geq c_1(M)^2.$$
This is usually called the Miyaoka-Yau inequality.

4. Assume that M is a Kähler manifold with the first Chern class $c_1(M)$ negative or zero. Then the equality in the Yau inequality holds if and only if the universal covering space of M is biholomorphic to the unit ball in $\mathbb{C}^n$ if $c_1(M) < 0$, and the complex vector space $\mathbb{C}^n$ if $c_1(M) = 0$. This gives a natural characterization of such important Hermitian locally symmetric spaces. For algebraic surfaces of general type, the Yau inequality was proved independently by Miyaoka [Mi1]. But even in this special case, the characterization of the equality case was not given there, and has not been proved by methods from algebraic geometry up to now.

5. If a complex surface M is homotopic to the complex projective plane $\mathbb{C}P^2$, then M is biholomorphic to $\mathbb{C}P^2$. This was conjectured by Severi. It also reminds one of the well-known Borel conjecture in geometric topology that two homotopic closed aspherical manifolds are homeomorphic. The proof of this result raised the problem of finding all fake projective planes. See the paper [PrY] for references and details.

6. The complex projective plane $\mathbb{C}P^2$ is the only simply connected algebraic surface with positive definite index.

7. If a Kähler manifold M is homeomorphic to the complex projective space $\mathbb{C}P^n$, then it is biholomorphic to $\mathbb{C}P^n$.

8. Let N be a compact complex surface that is a quotient of the unit ball in $\mathbb{C}^2$. Then any complex surface M that is oriented homotopically equivalent to N is biholomorphsically isomorphic to it. This reminds one of the Mostow strong rigidity, which in fact was used in the proof. Based on this, Yau conjectured that compact Kahler manifolds with negative sectional curvature is rigid if its deimension is greater than or equal to two. Yau suggested to use harmonic maps to settle this, and Siu [Siu80] carried this out in a special case with a stronger curvature assumption.

9. An easy consequence of Yau's solution to the Calabi conjecture gives the following structure theorem for Calabi-Yau manifolds: If M is a compact Kähler manifold with vanishing first Chern class, then there exists a finite covering of M which is the product of a complex torus with a simply connected Kähler manifold, whose first Chern class also vanishes.

10. Proof of the Torelli theorem for $K3$-surfaces [Tod1] together with the fact that every $K3$-surface is Kähler [Siu83] (the Torelli theorem for algebraic $K3$-surfaces [PiS], Kähler $K3$-surfaces [BuR] was known).

11. Proof of the surjectivity of the period map of $K3$-surfaces [Tod1] [Kul].

12. The moduli space of compact Calabi-Yau manifolds M is smooth, i.e., every element in $H^1(M,T)$ is tangent to a uniquely determined complex path of deformation of complex structures on M, and hence the moduli space of the complex structures on M have canonical coordinates.

13. Calabi-Yau threefolds M give rise to important examples of models of universe with spacetime supersymmetry, and the non-linear σ-model based on these manifolds give rises to a quantum cohomology group, i.e., a quantum deformation of the cohomology ring structure on $H^{1,1}(M)$. (See the articles [Leu1-2] [Lia] [Liu] [Tod2] in this volume.)

6 Harmonic maps

Harmonic maps are one of the analytical tools that have been systematically developed and applied by Yau to a wide range of questions in algebraic, Kähler and Riemannian geometry.

Let us recall some basic notations: Let $f\colon M \to N$ be a (smooth) map between Riemannian manifolds M and N. Let $e(f) =$ trace of the pull-back metric $f^*{ds_N}^2$ relative to the metric ${ds_M}^2$ on M and define $E(f) = \int_M e(f)$ as the energy of f. Harmonic maps then are minimizers, or more generally, critical points of this energy functional. The corresponding Euler-Lagrange equations constitute a quasi-linear elliptic system of second order. Thus, more generally, one can define harmonic maps also as the solutions of this system.

Harmonic maps work best when the target manifold has nonpositive sectional curvature because in that case, convexity properties of the distance function imply existence, uniqueness, and regularity results. Technical difficulties, however, arise in the case where the domain is noncompact. The problem essentially stems from the fact that the variational scheme that produces harmonic maps by minimizing the energy E, which is the most powerful method for producing harmonic maps, needs to have finite energy guaranteed because otherwise the space in which one seeks a harmonic map as the minimizer of the energy functional might be empty. Also, in order to obtain good estimates, be it for harmonic maps that are produced by a variational scheme or for the alternative parabolic heat flow method, one needs a finite energy condition. Such estimates then typically estimate higher order norms of a harmonic map in terms of its energy and geometric bounds on the manifolds involved. Many important contributions of Yau and his collaborators in the field are building upon a resolution of this difficulty, that is, producing harmonic maps of finite energy. This often required considerable technical ingenuity that went much beyond what other people had earlier achieved.

An example of this is already his first paper [76-1] in the field, written with Schoen, that proved the following basic existence result: Let M be a complete Riemannian manifold with non-negative Ricci curvature and N be a compact Riemannian manifold with non-positive sectional curvature, and let $f\colon M \to N$ be a map with $E(f) < \infty$. Then there exists a harmonic map $h\colon M \to N$ homotopic to f on each compact set in M and $E(h) < \infty$. With this tool at hand, the following result is proved: Let M be a complete Riemannian manifold with non-negative Ricci curvature and let D be a compact domain in M with smooth simply-connected boundary; then no nontrivial quotient group of $\pi_i(D)$ may appear as a subgroup of the fundamental group of any compact manifold with non-positive sectional curvature. It also applies the same idea to derive an analogous result when M is a complete non-compact stable minimal hypersurface in a complete non-negatively curved Riemannian manifold. Even when the latter is Euclidean space, this theorem was the first of its kind.

Besides the existence result for a harmonic map, the proof of the first result uses three other ingredients: (A) Uniqueness results for harmonic maps implied by the nonpositive curvature of N. (B) Every non-negative L^2-subharmonic function on a complete Riemannian manifold must be constant. (C) Every complete noncompact manifold of non-negative Ricci curvature has infinite volume.

As an application, it proves that if M be the complement of a point in $S^1 \times M'$, where S^1 is the circle and M' is any compact simply-connected manifold of dimension two or more, then M does not admit any complete metric of nonnegative Ricci curvature. This fact does not follow from other existing criteria for complete manifolds of nonnegative Ricci curvature.

The paper [78-5] with Schoen studies the following conjecture of Lawson and Yau: If f is a harmonic map between two compact Riemannian manifolds of negative curvature and f is a homotopy equivalence, then f is a diffeomorphism. The main result of the paper is the following theorem: Let M, N be compact Riemannian manifolds with $\dim M = \dim N = 2$, and genus ≥ 1; assume that N has nonpositive curvature; if $f\colon M \to N$ is a harmonic map of degree 1, then f is a diffeomorphism.

The proof uses certain refined Bochner type identities for the "holomorphic" ($\|\partial f\|^2$) and the "antiholomorphic" ($\|\bar{\partial} f\|^2$) part of the energy density of a harmonic map f With the help of these identities, the theorem is proved by establishing that the Jacobian of f has only isolated zeros which are nontrivial branch points of f, and then using a maximum principle argument. (A similar result was also found by Sampson [Sam78].) A generalization to the case of manifolds with boundary is given in the second half of the paper, using the continuity method.

The higher dimensional analogue of this result does not hold in the category of Riemannian manifolds. In fact, it was disapproved by Farrell, Ontaneda, and Raghunathan in [FaOR]. In the case of Kähler or locally symmetric manifolds, however, such a principle is valid, and this was at the basis of fundamental advances pioneered by Yau and described in the next two sections.

The first part of the paper [79-4] with Schoen proved the following result: Let M and N be compact and real-analytic Riemannian manifolds, and suppose that N has nonpositive sectional curvature. Let N_f be the set of all harmonic maps $M \to N$ homotopic to a harmonic map $f\colon M \to N$. Then N_f can be identified with a totally geodesic submanifold of N. If, in addition, $\pi_1(f)$ is surjective, then the homotopy class N_f is homeomorphic to the group of isometries generated by parallel translations of N. The proof uses the uniqueness properties of harmonic maps in a clever way. More precisely, in this context, harmonic maps are not quite unique, but all harmonic maps in the same homotopy class form a parallel family of maps of the same energy. Their image then yields the above totally geodesic submanifold of N. The real-analyticity of the spaces is needed to exclude that this parallel family has a boundary. From this result, several beautiful theorems about group actions on manifolds of nonpositive sectional curvature are derived.

The paper [79-8] summarizes and outlines the some work in [78-5] [76-1] and other topics including harmonic maps and the existence of incompressible minimal surfaces of positive genus in a compact Riemannian manifold, and an application of the uniqueness of harmonic maps to the study of groups acting on a manifold.

So far, harmonic maps had been considered between manifolds, but in the paper with Jost [91-1], harmonic maps between spaces that may also have quotient singularities were systematically studied. By lifting to universal covers, the problem is reduced to equivariant harmonic maps that have finite energy on some fundamental region. In particular, existence and uniqueness theorems for equivariant harmonic maps from a Riemannian manifold M to a symmetric space of noncompact type G/K are obtained. Equivariance here is with respect to the action of a discrete cocompact subgroup Γ of the automorphism group of M, and the action of G on G/K. They show that if $\rho\colon \Gamma \to G$ is a homomorphism, and if the Zariski closure of $\rho(\Gamma)$ is reductive, then there exists such an equivariant harmonic map. This generalizes earlier results of Corlette [Cor] and Donaldson [Do87b]; similar results were also obtained by Labourie [Lab]. In the case where M is Kähler and G is $\mathrm{SU}(n,1)$ or $\mathrm{SO}(n,1)$, it goes on to show that there exists an equivariant holomorphic or antiholomorphic map. This is relevant for the rigidity results described below.

The papers [91-3] and [93-1] with Jost discussed the role of harmonic maps in Kähler and algebraic geometry and describe various applications of harmonic maps to rigidity questions that will be discussed in the next two sections. [91-3] also included a criterion that a subvariety of a Kähler manifold cannot be blown down without creating a singularity, as well as the construction of pluriharmonic exhaustion functions on certain holomorphic fiber bundles.

The paper [94-6] with Shi establishes the existence of a harmonic map $f\colon M \to N$ when N is a compact manifold with nonpositive sectional curvature and M is a complete, noncompact manifold with nonnegative Ricci curvature. As already explained, the difficulty in proving this result lies in the noncompactness of the domain. The theorem is proved by evolving a smooth map with bounded energy density using the harmonic map heat flow, that is, by a parabolic instead of a variational method. Specifically, it uses of the existence of this flow on subdomains of M (having compact closure) with Dirichlet boundary conditions, which was established by Hamilton in [Ham3]. Then it shows that this parabolic flow has a smooth solution for all time on all of M. This is proved by establishing estimates on the derivatives of the solution which depend on the initial data but are independent of the domain, which follows from a Sobolev inequality for domains in M and then a Moser iteration argument together with the maximum principle.

7 Rigidity of Kähler manifolds

In the paper [76-6], Yau considers the question of when a complex manifold homotopic to a torus is biholomorphic to a complex torus. (Note that this true for Kähler manifolds by using the Albanese mapping). He proves two results: (1) a compact complex surface M with zero Euler number and a four-dimensional first real cohomology group that generates $H^4(M, R)$ under the cup product must be biholomorphic to a complex torus or have universal covering space biholomorphic to the disc times the complex plane, and further, that both situations can occur. (2) if M is a compact complex surface with topologically trivial real tangent bundle, then either (i) M has first Betti number equal to one, or (ii) M is a ruled surface of genus one, or (iii) M is covered by a complex torus or an elliptic fibre bundle over a curve of genus greater than one, or (iv) M is the quotient of C^2 by some volume-preserving affine group; in case (iv) the first Chern class of M is zero and the first Betti number is three. As a corollary, Yau constructs an example of a compact four dimensional parallelizable manifold, given by the product of a circle times the connected sum of the three dimensional real torus and the three dimensional real projective space, which does not admit any complex structure.

Utilizing a very deep suggestion of Yau, Siu [Siu-80] had obtained strong rigidity results for Kähler manifolds under certain negativity or nonpositivity conditions on their curvature tensor. These conditions are stronger than negative (nonpositive) sectional curvature, but include Hermitian symmetric spaces of noncompact type. Whereas the famous rigidity results of Mostow and Margulis had been valid in the category of symmetric spaces, that is, two homotopically equivalent compact locally symmetric spaces of noncompact type other than hyperbolic Riemann surfaces had to be isometric to each other, Siu's result worked in the category of Kähler manifolds. Under his curvature conditions (which again exclude hyperbolic Riemann surfaces), two homotopically equivalent compact Kähler manifolds have to be biholomorpic (or antibiholomorphic) to each other. Siu's method was to show that a harmonic homotopy equivalence between such Kähler manifolds which exists by the general existence theory has, in fact, to be (anti)biholomorphic. We see here an important general principle, namely that under appropriate circumstances, a harmonic map has to satisfy stronger properties that tightly reflect the geometry of the spaces involved.

As indicated, this work was inspired by Yau, and it was also the starting point of a fundamental development in Kähler geometry and symmetric space theory. Again, Yau and his collaborators played a leading role in this development.

One of the cases not covered by Siu's method are quotients of polydiscs. Such quotients include products of Riemann surfaces, but there also exist compact quotiens that are irreducible, that is, not finitely covered by a product of Riemann surfaces. Siu's scheme does not work here because harmonic maps in this context need not be holomorphic – after all, hyperbolic Riemann surfaces can be deformed, that is, carry different holomorphic structures with the same underlying topology. The paper with Jost [83-3] therefore develops a method of studying compact Kähler manifolds which admit mappings of maximal rank everywhere into quotients of polydiscs, i.e., into Riemann surfaces or products of them. The main tool is a detailed investigation of the harmonic mappings in the corresponding homotopy classes. This paper then made an important discovery that was basic for subsequent work of many people on Higgs bundles, Shafarevitch type classification problems for Kähler manifolds etc. The discovery is that the local level sets of harmonic mappings in this context are analytic subvarieties of the domain, even though the harmonic map need not be holomorphic itself. (For example, this result implies that for a compact Kähler manifold to admit a (nontrivial) holomorphic map to a hyperbolic Riemann surface is a topological property. More precisely, we have the following result: Let M be a compact Kähler manifold, and $g\colon M \to S$ a continuous map to some compact Riemann surface of genus ≥ 2. Assume g is nontrivial on the second homology. Then there exists a nonsingular holomorphic curve C of genus ≥ 2 and a nontrivial holomorphic map $h\colon M \to C$. In particular, the universal cover of M admits a bounded holomorphic function.)

The paper then goes on to derive conditions on the Chern and Kähler classes of the manifolds under which the harmonic map is of maximal rank everywhere and, in the case the domain and image have the same dimension, in particular a local diffeomorphism. In the latter case, the condition can be formulated as follows:

$$c_1(M) \cup \Omega^{m-1}[M] = f^* c_1(N) \cup \Omega^{m-1}[M],$$

where $[M]$ denotes the fundamental homology class of the domain M, $\Omega[M]$ its Kähler class and $c_1(M)$ its first Chern class. The image N is always assumed to be a compact quotient of polydiscs with the usual induced metric and complex structure; moreover, the functional determinant of $f\colon M \to N$ is not identically zero. Under the same condition, the paper shows that a compact Kähler manifold which is homeomorphic to a quotient of polydiscs must also be such a quotient. Thus, in the product case, the only possible deformations even in the context of Kähler manifolds are the obvious ones, that is, those arising from deforming the factors.

Another important application of the method is to show that every deformation of of a Kodaira surface is again such a surface. (Kodaira surfaces were introduced by Kodaira and also independently by Atiyah to provide examples of algebraic surfaces with positive index and of fibre spaces whose signature is different from the product of the signatures of the base space and the fiber. They can also be considered as locally nontrivial families of Riemann surfaces). Consequently, the moduli space of a Kodaira surface M is the moduli space of the Riemann surface underlying the construction of M. The local version of this result was proved Kas [Kas].

Globalizing the above deformation results, the paper with Jost [85-4] shows that if D is the unit disc and Γ is an irreducible subgroup of $\mathrm{Aut}(D \times D)$ with compact quotient, then the complex structure of $N = D \times D/\Gamma$ is strongly rigid, i.e., if M is a compact complex surface homotopy equivalent to N, then M is biholomorphic to N. Again, this is stronger than Mostow strong rigidity [Mos] for N in that the complex structure of M is arbitrary but not necessarily Hermitian locally symmetric. The higher dimensional version of this result was then shown by Mok [Mok85].

To prove this rigidity result, Jost and Yau show that M algebraic, in particular Kähler, using Kodaira's classification of surfaces, and then deform the homotopy equivalence to a harmonic map f. Then the results of [83-3] can be applied to reach the conclusion.

Since those results essentially completed the rigidity question in the Kähler category in the compact case, Jost and Yau then turned to the finite volume. As already explained in Section 6, this case is much more difficult because of the problems with finite energy harmonic maps.

The paper with Jost [86-5] deals with locally symmetric complex manifolds of rank one with finite volume and their strong rigidity within the class of Kähler manifolds admitting good compactifications. As indicated, the significant analytical achievement and starting point is the construction of a proper harmonic homotopy equivalence of finite energy and maximal rank. The strong rigidity of an irreducible quotient H^n/Γ is also proved, where $H^n, n \geq 2$, is the product of upper half planes and Γ is a discrete irreducible torsion-free subgroup of $\mathrm{Aut(H^n)}$. This then is the extension to the finite volume case of the results just described in the compact case.

More precisely, let $\overline{M}$ be a compact Kähler manifold, S be a subvariety of $\overline{M}$ with normal crossings, and consider $M = \overline{M} - S$. If M is properly homotopically equivalent to a quotient $N = D/\Gamma$ of finite volume, where D is an irreducible bounded symmetric domain in $\mathbb{C}^n$, $n \geq 2$, of rank one and Γ is a discrete torsion-free subgroup of $\mathrm{Aut(D)}$, then M is biholomorphic or antibiholomorphic to N. The second result states that if M is properly homotopically equivalent to $N = H^n/\Gamma$, then there exists a diffeomorphism $f\, M \to N$ whose lift $F\, \tilde{M} \to H^n$ to the universal covering has holomorphic or antiholomorphic coordinate components.

The paper [87-11] with Jost proves the strong rigidity of finite volume quotients of irreducible bounded symmetric domains and the action of the mapping class group acting on the Teichmüller space. Specifically, it proves two results:

(1) Let D be an irreducible bounded symmetric domain in $\mathbb{C}^n$, $n \geq 2$. Let Γ be a discrete subgroup of $\mathrm{Aut\,D}$ for which D/Γ is noncompact and has finite volume. Suppose that a group $\tilde{\Gamma}$ acts as a discrete automorphism group on a contractible Kähler manifold $\tilde{M}$. Assume that $\tilde{M}/\tilde{\Gamma}$ admits as a finite cover a quasiprojective manifold M and that $\overline{M} - M$ is of codimension at least three in $\overline{M}$. If $\tilde{\Gamma}$ is isomorphic to Γ (as an abstract group), then $\tilde{M}$ is (anti)biholomorphically equivalent to D, and $\tilde{\Gamma}$ is conjugate to Γ in $\mathrm{Aut\,D}$.

(2) Let T_g be the Teichmüller space of Riemann surfaces of genus g, Γ_g the Teichmüller modular group and $N = \mathcal{M}_g$ the moduli space, $g \geq 2$. Let $\hat{\Gamma}$ be isomorphic to Γ and act on a contractible Kähler manifold $\tilde{M}$ with a quotient $\tilde{M}/\hat{\Gamma}$ satisfying the same assumptions as in (1). Then $\tilde{M}$ is biholomorphically equivalent to T_g, and $\hat{\Gamma}$ acts as the Teichmüller modular group.

The proof uses again finite energy harmonic maps. The condition on the codimension of $\overline{M} - M$ is needed for constructing some finite energy map in the given homotopy class which can then be deformed into a finite energy harmonic map. The finite energy condition was later on removed by Jost-Zuo [JoZu] who were able to construct a harmonic map of not necessarily finite energy that could nevertheless be controlled tightly enough so that a subtle local analysis in the sense of [Siu80] can be applied. Thus, also the restriction on the dimension of the boundary of M in its compactification could be removed and a full generalization of the results of Margulis from the category of Hermitian locally symmetric spaces to the ones of Kähler manifolds be achieved.

The paper [91-9] with Zheng characterizes compact Kähler manifolds M^n, $n \geq 2$, which admit negatively $\frac{1}{4}$-pinched Riemannian metrics. On a compact Kähler surface M^2, if it admits a negatively $\frac{1}{4}$-pinched Riemannian metric ds^2, (i.e., the sectional curvature is bounded between -1 and $-\frac{1}{4}$, then it is not hard to show that $3\tau \geq \chi$, where where τ, χ are the signature and Euler number of M, respectively. In terms of Chern numbers, this is just $c_1^2(M) \geq 3c_2(M)$. Since the curvature of M is negative, it is a surface of general type with $c_1(M) < 0$. By [77-1], $c_1^2(M) = 3c_2(M)$, and it follows that M^2 is covered by the unit ball $B^2 \subseteq \mathbb{C}^2$, i.e., M^2 is biholomorphic to some B^2/Γ. For higher dimensions, one can ask a similar question. The paper [91-9] gives an affirmative answer to it. The same result was obtained independently by Hernandez in [Her].

The proof uses the method of Sampson [Sam] where he proved that there is no minimal immersion from a compact Kähler manifold into a Riemannian space with constant negative curvature. The method is also applied to study studies harmonic maps from a compact Kähler manifold into a negatively δ-pinched Riemannian manifold with $\delta > \frac{1}{4}$. The finite volume case was later treated by Jost-Yang [JoYa].

The paper [93-14] with Zheng introduces the definition of a semirigid manifold and studies the class of semirigid general type manifolds. Let M be a compact complex manifold and $\mathcal{C}(M)$ the space of all Kähler metrics on M. Let $\mathcal{F}(M)$ be the space of all Kähler metrics on M with nonpositive holomorphic bisectional curvature. M is said to be semirigid if $\mathcal{F}(M)$ is not empty, and its linear span in $\mathcal{C}(M)$ is finite-dimensional. Semi-rigidity is likely to occur only when the cotangent bundle T^*M is semi-ample but not ample. For example, any complex torus is semirigid.

The class of semirigid n-dimensional compact Kähler manifolds (M, g) studied here satisfies the following conditions: (1) $n \geq 2$, and $(c_1^2 - c_2) \cdot [\omega_h]^{n-2} = 0$ for a Kähler metric h; (2) $g \in \mathcal{F}(M)$; (3) $\{x \in M\colon \mathrm{Ric}_g^n(x) \neq 0\}$ is dense in M, and $\{x \in M\colon \mathrm{Ric}_g^{n-1}(x) \neq 0\}$ is a Zariski open subset in M. Under these conditions, then there exists an isometric holomorphic immersion $f : (\tilde{M}, \tilde{g}) \to (\mathbb{C}^{n+1}, g_0)$ from the universal covering space of (M, g) into the complex euclidean space, and for each $\gamma \in \pi_1(M)$, there is a rigid motion ϕ_γ in $\mathbb{C}^{n+1}$ such that $f \circ \gamma = \phi_\gamma \circ f$. Another result states that for any $h \in \mathcal{F}(M)$, (M, h) also satisfies the condition (3); and for any two isometric holomorphic immersions $f : (\tilde{M}, \tilde{g}) \to (\mathbb{C}^{n+1}, g_0)$ and $f^{(h)} : (\tilde{M}, \tilde{g}) \to (\mathbb{C}^{n+1}, g_0)$, there always exists an affine transformation ϕ in $\mathbb{C}^{n+1}$ such that $f^{(h)} = \phi \circ f$.

From the differential-geometric point of view, the first result says that any manifold M in the above class, locally (M, g) looks a hypersurface in $\mathbb{C}^N$; and the second result says that for any two metrics in $\mathcal{F}(M)$, their holomorphic bisectional curvature tensors are conformal to each other.

Some earlier related results were obtained by Mok in [Mok].

So far, the methods and results only apply in the context of Kähler manifolds; the essential reason that the Kähler condition expresses a compatibility between a Riemannian and a complex structure. In particular, this implies that holomorphic maps between Kähler manifolds are harmonic, and the achievements just described then are concerned with the reverse direction, namely to show that, under appropriate circumstances, harmonic maps have to be holomorphic. Clearly, this scheme seems to break down for general Hermitian manifolds that are not Kähler. Therefore, in order too study maps from Hermitian to Riemannian manifolds, the paper [93-4] with Jost introduces and studies the new nonlinear elliptic system of equations

$$\gamma^{\alpha\overline{\beta}}\left(\frac{\partial^2 f^i}{\partial z^\alpha \partial z^{\overline{\beta}}} + \Gamma^i_{jk}\frac{\partial f^j}{\partial z^\alpha}\frac{\partial f^k}{\partial z^{\overline{\beta}}}\right) = 0, \quad i = 1, \cdots, \dim N,$$

where f is a map from a complex manifold X with Hermitian metric $(\gamma_{\alpha\overline{\beta}})$ in local coordinates into a Riemannian manifold N with metric (g_{ij}) and Christoffel symbols Γ^i_{jk}. This new system is equivalent to the harmonic map system if X is Kählerian. But, in general, the system is analytically more difficult than that of harmonic maps, because it neither has a divergence nor a variational structure. However, the system is more appropriate for Hermitian geometry than the harmonic map system because in contrast to the latter, it is always satisfied by holomorphic maps. A solution to the new system is called Hermitian harmonic.

The paper treats the existence problem to the above system by a parabolic method, solves the Dirichlet problem, and gives applications of the existence result to complex geometry. For example, it extends Siu's rigidity theorems in [Siu80] to the case where the manifold M compared with the Hermitian locally symmetric space is only astheno-Kählerian, where an m-dimensional Hermitian manifold X with Hermitian metric $\gamma_{\alpha\overline{\beta}}dz^\alpha dz^{\overline{\beta}}$ is called astheno-Kähler if $\omega = \frac{1}{2}i\gamma_{\alpha\overline{\beta}}dz^\alpha \wedge dz^{\overline{\beta}}$ satisfies $\partial\overline{\partial}\omega^{m-2} = 0$ (this corrected definition was given in [93-4]). Hence, without using Kodaira's classification of compact complex surfaces, it shows that a compact complex surface homotopy equivalent to a quotient of the unit ball in $\mathbb{C}^2$ is already $\pm$ biholomorphically equivalent to this quotient. Also, without using either Kodaira's results or Donaldson's theory of differential structures on 4-manifolds, it proves that if N is a compact quotient of the unit ball in $\mathbb{C}^2$ (without singularities) and M is a 4-manifold with nontrivial fundamental group, then the connected sum of N and M cannot be homotopy equivalent to a complex surface.

The paper [93-3] proves the following splitting theorem: Let M be an n-dimensional projective manifold and $D \subset M$ be a divisor with normal crossings. Let K_M be the canonical divisor of M. Suppose that $K_M + D$ is nef, big and ample modulo D. Suppose the bundle $\Omega_M(\log D)$ admits an endomorphism which has nontrivial kernel somewhere in $M - D$. Then $M - D$ is covered by the product of two manifolds M_1 and M_2 so that $M - D$ is biholomorphic to the quotient of $M_1 \times M_2$ by a discrete group of automorphisms which preserve the product structure of $M_1 \times M_2$.

A consequence of this result gives an algebraic geometric characterization of locally Hermitian symmetric spaces of rank at least 2 [93-3, Theorem 2.3].

These results follow from the completeness of the almost complete Kähler-Einstein metric on a quasiprojective manifold $M-D$ constructed earlier in [87-8] under the condition that K_M+D is nef, big and ample modulo D. In addition to Kähler manifolds, some splitting theorems are obtained for Hermitian manifolds with admissible Hermitian metrics (i.e. a Hermitian metric whose Kähler form satisfies $\partial\overline{\partial}\omega^{n-1} = 0$), by the argument replacing the Kähler-Einstein metric by an Hermitian Yang-Mills connection over TM (if it exists). Concerning the existence of a Hermitian Yang-Mills connection, the existence theorem on a stable bundle over a compact complex manifold with an admissible Hermitian metric [87-3] [Buc] is generalized to the following two cases with suitable definitions of stability: (1) a holomorphic vector bundle $V = \tilde{V}/\Gamma$ over a compact complex manifold $M = \tilde{M}/\Gamma$, where Γ is a subgroup of G acting on $(\tilde{M}, \tilde{V})$ with a G-invariant admissible Hermitian form and a G-equivariant Hermitian connection; (2) a holomorphic vector bundle over $X - D$ which is extendable over D by orbifold, where D is a normal crossing divisor of a complex manifold X. Here a bundle over $X - D$ is called extendable over D by orbifold if there is an open covering of a neighbourhood of D in X, say $\{U_i\}$, and ramified covering $\pi_i\colon \tilde{U}_i \to U_i$ such that the pull-back bundle $\pi_i^* V$ over $\tilde{U}_i - \tilde{D}_i$ ($\tilde{D}_i = \pi_i^{-1}(U_i \cap D)$) is extendable as a holomorphic vector bundle and two nearby extensions can be pieced together nicely.

In [91-1], Jost and Yau showed the following result: Let M be a compact Kähler manifold. Then $\pi_1(M)$ not isomorphic to the fundamental group of a (real) hyperbolic space form of dimension >2. In other words, the topology of Kähler manifolds and real hyperbolic space forms is completely different – except of course in real dimension 2 where Riemann surfaces of higher genus are both hyperbolic space forms and Kähler manifolds. Also, we have the following result: Let G/K be an irreducible Hermitian symmetric space of noncompact type with $\mathrm{rank}(G/K) \geq 2$. Let Γ be a cocompact lattice. Then every discrete homomorphism $f\colon \Gamma \to \mathrm{SO}(n,1)$ or $\mathrm{SU}(n,1)$ is finite. Similar results were obtained by Carlson-Toledo.

In [93-15], Jost and Yau applied uniqueness and holomorphicity properties of harmonic maps and Yau's general Schwarz lemma to classical problems in algebraic geometry, the Mordell and Shafarevitch problems over functions. They developed new proofs and substantial generalizations of the following resolutions of those problems:

Theorem (Manin, Grauert). *Let C be a nonsingular holomorphic curve, $\Sigma(t)$ a curve of genus ≥ 2 over the function field of C. If $\Sigma(t)$ is not isotrivial, then it has at most finitely many rational points.*

Theorem (Arakelov, Parshin). *Let C be a nonsingular holomorphic curve, and $S \subset C$ a finite subset. Then there exists at most finitely many nonisotrivial curves of given genus $g \geq 2$ over the function field of C with bad reduction at most over S.*

They also derived various existence results for such fibrations.

8 Super-rigidity of spaces of nonpositive curvature

After the rigidity theory of Kähler manifolds had attained a rather satisfactory and complete state, by the results described in Section 7, the question emerged whether harmonic maps can also contribute to rigidity theory in the wider context of Riemannian manifolds. Clearly, Riemannian metrics of, say, negative sectional curvature in general are not rigid, and so, the first task was to identify the appropriate categories. One natural category of course is the one of locally symmetric spaces of noncompact type, as follows from the fundamental work of Mostow and Margulis. Therefore, a guiding question was to what extent harmonic maps can be used to rederive and generalize the rigidity theorem of Mostow and the superrigidity theorem of Margulis.

In that direction, a very general solution was obtained in the paper [93-5] with Jost (and also in Mok-Siu-Yeung [MokSY]) that uses harmonic maps to prove super-rigidity of certain compact locally symmetric spaces. One result of the paper states: Let $\widetilde{M} = G/K$ be an irreducible symmetric space of noncompact type, other than $\mathrm{SO}_0(p,1)/\mathrm{SO}(p) \times \mathrm{SO}(1)$, $\mathrm{SU}(p,1)/\mathrm{S}(\mathrm{U(p)} \times \mathrm{U}(1))$. Let Γ be a discrete cocompact subgroup of G (also called a cocompact lattice). Let $\widetilde{N}$ be a complete simply connected Riemannian manifold of nonpositive curvature operator with isometry group $I(\widetilde{N})$. Let $\rho\colon \Gamma \to I(\widetilde{N})$ be a homomorphism for which $\rho(\Gamma)$ either does not have a fixed point on the sphere at infinity of $\widetilde{N}$ or, if it does, it centralizes a totally geodesic flat subspace. Then there exists a totally geodesic ρ-equivariant map $f\colon \widetilde{M} \to \widetilde{N}$.

A consequence of the above result is the following: Let $\widetilde{M} = G/K$ and Γ be as above. Let H be a semisimple noncompact Lie group with trivial center, $\rho\colon \Gamma \to H$ a homomorphism with Zariski dense image. Then ρ extends to a homomorphism from G onto H.

In fact, this result not only includes the Margulis results that are valid for the case of rank at least 2, but also the ones of Corlette [Cor91] for quaternionic hyperbolic spaces and of Gromov and Schoen [GrS] for the hyperbolic Cayley plane that had also been obtained by harmonic map techniques. Also, harmonic map theory had been extended to deal with maps into metric spaces of nonpositive curvature in some generalized sense, like Euclidean Bruhat-Tits buildings; these spaces no longer carry a Riemannian, and actually not even a smooth structure, but this could be overcome with the help of rather general variational principles, see e.g. [Jo].

[93-5] then was carried out in that generality and therefore also extends the corresponding result of Gromov and Schoen [GrS] as follows: Let $\widetilde{M} = G/K$ and Γ be as above. Let $\rho\colon \Gamma \to \mathrm{SL}(n, \mathbf{Q}_p)$ be a homomorphism, for some $n \in \mathbf{N}$ and some p. Then $\rho(\Gamma)$ is contained in a compact subgroup of $\mathrm{SL}(n, \mathbf{Q}_p)$. The conclusion of this result holds more generally for a homomorphism ρ from Γ into the isometry group of an F-connected complex in the sense of Gromov and Schoen.

In a somewhat different direction, more precisely in the one of local product structures, the paper with Jost [99-6] makes use of harmonic map techniques to show certain rigidity properties of nonpositively curved spaces. A remarkable feature of this work is that the authors consider rigidity and splitting properties for nonpositively curved metric spaces (in the sense of Aleksandrov from now on) instead of only Riemannian manifolds.

Specifically, it proves the following result: if M is a compact metric space of nonpositive curvature, whose fundamental group Γ is center-free and splits as a product $\Gamma_1 \times \cdots \times \Gamma_k$, then M is isometric to a product $X_1/\Gamma_1 \times \cdots \times X_k/\Gamma_k$, where each X_i is a complete simply connected metric space of nonpositive curvature acted upon by Γ_i isometrically.

To prove this, it first considers a product $M_1 \times \cdots \times M_k$ where each factor M_i is a compact $K(\Gamma_i, 1)$ space, and a homotopy equivalence $F\colon M_1 \times \cdots \times M_k \to M$. Each realization of M_j as a factor of the product $M_1 \times \cdots \times M_k$ provides from F a map $M_j \to M$ which is deformed (with the use of the first author's previous work, see [Jo]) into a harmonic map f_{s_j} that depends on the particular realization of M_j as a factor. A uniqueness result described in the paper is then used to build suitable foliations on the universal cover of M which are proved to induce the required splitting.

It also shows that if a group Γ acts freely, properly discontinuously and irreducibly on a nontrivial product Riemannian manifold X with compact quotient, and such quotient X/Γ is homotopically equivalent to a compact Riemannian manifold M with nonpositive curvature, then X and the universal cover of M are homogeneous and there is a totally geodesic homeomorphism between X/Γ and M.

9 Survey papers by Yau

Survey papers have played an important role in introducing people into new fields and keeping experts updated about exciting new developments. Besides publishing many research papers, Yau has also written many survey articles. We mention some of them here.

[80-7] was based on his Plenary talk at ICM in 1978 and gave an extensive survey about problems in differential geometry and their associated nonlinear partial differential equations. Yau discussed status and his visions for the following themes: the existence of complete Riemann metrics with prescribed sign of the scalar curvature and Ricci curvature, the positive mass conjecture in general relativity, the questions related to the Monge-Ampère equation, and the existence of complete Kähler-Einstein metrics with constant (or zero) Ricci curvature on both compact and non-compact complex manifolds.

The paper [82-7] is a survey article in the very influential book [82-11] which was based on the special year organized by Yau at IAS, princeton, 1979-1980. In this article of more than 60 pages, Yau gave a survey of analytic methods in differential geometry based on partial differential equations.

It covered the following topics: isoperimetric inequalities and inequalities of Poincaré and Sobolev types; harmonic functions and eigenfunctions on complete Riemannian manifolds; heat equations and the wave equation; isometric deformations of surfaces in $\mathbb{R}^3$; scalar equations whose higher derivatives enter linearly; the minimal surface equation; equations whose higher derivatives enter nonlinearly (Monge-Ampére type); harmonic forms and the Dirac equation; the $\overline{\partial}$-operator on manifolds; harmonic maps; the Yang-Mills equations; minimal submanifolds of higher codimension; and isometric immersions. Yau also stated some unsolved problems.

The article [87-4] is the written version of a plenary talk by Yau at the GR 11 meeting in Stockholm in 1986 and described some of the then recent developments in understanding global structure in general relativity. It discussed briefly work by Christodoulou on weak solutions of the spherically symmetric Einstein equations, and of the proof by Schoen and Yau of the positivity of the ADM mass and its relevance to the question of the global existence of solutions.

The paper [91-2] discussed the progress on Yau's proposed generalization of the classical uniformization to high dimension 15 years earlier. It considers three cases: elliptic, parabolic and hyperbolic.

The paper [93-2] with Hübsch gave an overview, from a mathematical standpoint, of $(2, 2)$ superconformal field theories and the relation with different mathematical aspects, such as quantum cohomology, Lefschetz-like decompositions of certain Jacobian rings, counting (rational) curves of given degree on a Calabi-Yau threefold, etc.

In the paper [96-7], Yau surveyed a vast range of topics related to the existence of Kähler-Einstein metrics and consequences: (1) his attempt to construct counter-examples by an analogue of the Hitchin inequality for 4-dimensional Einstein manifolds, (2) inequalities between Chern numbers, (3) Kähler-Einstein metrics on noncompact manifolds, (4) explicit construction of a Kähler-Einstein metric, an example being the Eguchi-Hanson metric, (5) connection with semistability, (6) smoothness of the moduli space of Calabi-Yau manifolds, along with its relation to conformal field theory, (7) effective version of the base-point-free theorem, Fano manifolds.

In the paper [99-15], Yau gave a survey of the areas of mathematics and physics surrounding Calabi-Yau metrics and mirror symmetry. It covered the following topics: (1) the work of Tian and Yau on Ricci-flat metrics on complete Kähler manifolds, (2) the algebro-geometric consequences of the existence of Ricci-flat metrics, (3) and the existence and study of hyper-Kähler metrics, (4) string theory and the discovery of mirror symmetry, (5) the calculations of P. Candelas et al. predicting numbers of rational curves on the quintic which sparked new eras in algebraic geometry and string theory, (6) the resulting work on defining and calculating quantum cohomology, (7) the SYZ conjecture providing a conjectural geometric basis for mirror symmetry, (8) exceptional holonomy, in particular the work of D.Joyce, (9) and curve counting in Kähler surfaces.

He also mentioned many open problems, with suggestions for the future direction of the field, for example, the right geometric framework (canonical metrics, special Lagrangian cycles, etc.) for dealing with the "quantum geometry" of ordinary double points and flops.

The paper [00-13] (or [00-16]) gave the historic background, current trends and many open questions in differential geometry which were considered important by Yau for the future development of the subject. It covered the following topics: (1) submanifolds, natural questions which arise when studying how to embed a Riemannian surface into $\mathbb{R}^3$, and also embedding problems in other dimensions and spaces, (2) intrinsic geometry, relations between the geometric properties such as the full curvature tensor, the Ricci tensor and the scalar curvature and the topology of the manifolds, (3) Einstein metrics, (4) ten important open problems in the topics in this survey.

The article [05-9] with Liu and Sun surveyed the role of differential geometry and geometric analysis in exposing the structure of moduli spaces of Riemann surfaces and their associated

Teichmüller spaces from the perspective of ten metrics: (1) three Finsler metrics, defined by Teichmüller, Kobayashi and Carathèodory, and extensively deployed in understanding the hyperbolic geometry of moduli spaces and mapping class groups, (2) seven Kähler metrics defined on the moduli space, the Weil-Petersson, the Cheng-Yau Kähler-Einstein, the McMullen, the Bergman, and asymptotic Poincaré metrics, where numerous results pertaining to curvature, volume, injectivity radii and L^2-cohomology over the moduli space were summarised within the framework of these metrics, (3) two new metrics introduced in their recent work, the Ricci metric and perturbed Ricci metric, the first coming from the negative Ricci form of the Weil-Petersson metric and the latter being a combination of the Ricci and Weil-Petersson metrics. The curvature properties of the perturbed Ricci metric allowed them to establish equivalence among the nine complete metrics (i.e., excluding the Weil-Petersson metric). They also sketched a proof of positivity for the first Chern class and Mumford stability of the logarithmic cotangent bundle on the moduli space.

In the article[05-10], Yau reviewed the brief history of complex geometry and looked into the future directions. He discussed almost all the important topics in complex geometry: Riemann surfaces, the uniformization theorem, Teichmüller space, Kähler geometry, Hodge structure, classification of complex manifolds, Kähler-Einstein metrics, harmonic maps, deformation of complex structures, moduli space of polarized algebraic manifolds, and Calabi-Yau manifolds. He also raised some important open questions.

In the paper [02-4] Yau surveyed some developments of the geometry and physics of Einstein manifolds: (1) the concept of the holonomy group of Riemannian manifolds, (2) recent developments of the mirror symmetry conjecture for Calabi-Yau threefolds, including its consequences in algebraic geometry, (3) the geometry of G_2-manifolds (4) recent works on the general relativity on four-dimensional Lorentzian manifolds, including the Einstein-Dirac-Yang-Mills system, black holes and horizons.

In a recent extensive survey [07-4], Yau gave a rather complete historical survey of the field of geometric analysis as one of the main players of this important field. He gave many important observations and philosophical points of views on both problems and methods employed to attack them. Perhaps the following headings of the sections will give the sense of the large scope of this article: (1) History and contributors of the subject , (2) Construction of functions in geometry, (3) Mappings between manifolds and rigidity of geometric structures, (4) Submanifolds defined by variational principles, (5) Construction of geometric structures on bundles and manifolds. This survey included an impressive list of 755 references.

The most recent surveys include [09-10] and [09-8]. [09-10] gives a comprehensive introduction to the history and results on the Calabi-Yau manifolds, recent developments and their applications. It is perhaps the best expository reference on this very important class of Kähler manifolds. [09-8] is published in the popular Bulletin of Amer. Math. Soc. and gives a historical introduction to general relativity and black holes, and then a survey of the Kerr geometry and wave dynamics in the Kerr space-time geometry and a rigorous mathematical description of some of the key physical properties of black holes.

10 Open problems by Yau

Yau has made and suggested generously many open problems on various occasions. We mention and briefly comment on several of them from different lists in order to convey a sense of their depths and impacts.

The first and also most famous list of open problems by Yau was published in [82-9]. It contained 120 open problems in many different aspects of differential geometry. As Yau explained in the beginning, "*The difficulty of the problems ranges from "elementary" to "deep." Whereas "deep" problems may be solved by a beginning student within a few months and elementary problems can be open for a long time, I do hope that this problem set will provide a condensed overview of the subject for beginning students*".

The topics covered in this set are broad and clearly reflected the wide range of interests of Yau.[2] Yau has worked on many topics related to them, and this problem has indeed provided an overview of geometric analysis to both experts and beginning students.

Many problems in this set are supplied with commentaries and references. The problems are grouped in the following sections: I. Curvature and the topology of manifolds (A. Sectional curvature. B. Ricci curvature. C. Scalar curvature, 32 problems in all), II. Curvature and complex structure (17 problems), III. Submanifolds (17 problems), IV. The spectrum (14 problems), V. Problems related to geodesics (7 problems), VI. Minimal submanifolds (26 problems), VII. General relativity and the Yang-Mills equation (7 problems).

Some of the problems were well-known or due to others. But many were raised by Yau. We mention several such problems here.

Problem 34 [82-9]. *Let M be a complete, noncompact, Kähler manifold with positive bisectional curvature. Prove that M is biholomorphic to $\mathbb{C}^n$.*

It is not even known if this manifold is Stein. If the sectional curvature is positive, then M is Stein as was observed by Greene and Wu [GrW4]. For geometric conditions which guarantee manifolds to be $\mathbb{C}^n$, see Siu-Yau [77-4].

There has been a lot of work on Problem 34 [82-9]. Though it is not completely solved yet, many partial results have been obtained. For example, Chen, Tang and Zhu [ChTZ] proved that a complete non-compact complex two-dimensional Kähler manifold M of positive and bounded holomorphic bisectional curvature whose its geodesic balls have maximal volume growth is biholomorphic to $\mathbb{C}^2$. Chau and Tam [ChaT1] [ChaT2] proved that a complete noncompact Kähler manifold with bounded nonnegative holomorphic bisectional curvature and maximal volume growth is biholomorphic to $\mathbb{C}^n$. They also proved that the maximal volume growth condition can be removed if the Kähler manifold has average quadratic scalar curvature decay and positive curvature operator. See the survey paper by Tam [Tam] for the status of this conjecture and related references.

Problem 65 [82-9]. *Can one define the rank of a compact C^ω-manifold M with nonpositive curvature so that if M is a locally symmetric space space the definition agrees with the standard one? Suppose that there is a totally geodesic immersed, flat 2-plane in M. Can one find an immersed totally geodesic torus in M? If the "rank" of M is greater than one, one expects that M is very rigid. How do we describe this rigidity?*

This problem 65 in [82-9] has been completely settled. For the definition of rank and higher rank rigidity, see the papers [Bal] [BuS] and many references there. For the existence of closed flats, see [BaS]. The efforts to solve this problem have completely changed the subject of manifolds of nonpositive curvature. Besides the papers [Bal] [BuS] and [BaS], see also the books [BaGS] [Ebe] and many references there.

[2] According to Yau, he worked intensively for three weeks to produce this list of open problems.

The paper [91-2] also contains several problems related to the uniformization of complex manifolds in higher dimensions of all three types: compact Kähler manifolds with nonnegative bisectional curvature (a generalized Frankel conjecture); Kähler manifolds of parabolic type; Kähler manifolds of hyperbolic type.

The list [92-11] (or [93-6]) is a new list of 100 open problems and complemented the earlier list [82-9]. Unlike the list [92-11], most of the problems here were raised by Yau. As Yau said, *"a field is active if it has many interesting open problems."* The problems are classified according to five topics: I. Metric geometry; II. Classical Euclidean geometry; III. Partial differential equations; IV. Kähler geometry.

The following is an important problem from this list.

Problem 65 [92-11]. *Prove that a compact Kähler manifold with positive first Chern class admits a Kähler metric if and only if the manifold is stable in the sense of geometric invariant theory, the tangent bundle is stable as a bundle and the automorphism group is reductive....*

This conjecture was actually formulated by him in 1980s at the Institute for Advanced Studies, Princeton, and explained at various seminars. It was first formally published in [86-6] (or [87-7] on p. 142):

"So far, however, all examples of a Kähler manifold M with positive first Chern class which does not admit a Kähler-Einstein metric admit nontrivial holomorphic vector field, it is natural to ask the following question: If there exists no nonzero holomorphic vector field on M, and if the tangent bundle of M is stable, can we always minimize the functional F? The motivation for the assumption on the stability will be discussed later. Of course, if the answer to the above question is yes, then (*) would also be a sufficient condition for the existence of Kähler-Einstein metrics."

The motivation mentioned above probably referred to the discussions on Hermitian-Einstein metrics on stable bundles in the same paper [87-7], pp. 144-145.

Note that the functional F is defined as follows. Fix a Kähler class Ω on M, and let H_Ω be the space of all Kähler metrics with Kähler class equal to Ω. For every metric $g \in H_\Omega$, let R be its scalar curvature. Then F is defined by

$$F : H_\Omega \to \mathbb{R}, \quad g \mapsto \int_M R^2.$$

A critical point of this functional is called an extremal metric by Calabi, and any Kähler-Einstein metric is an extremal metric.

There has been a lot of work related to this conjecture, for example, this conjecture has been made more precise and extended to study Kähler metrics of constant scalar curvature and many different notions of stability of manifolds have been used or introduced. There are also many papers on studying the asymptotic behaviors of the Ricci flow on such Kähler manifolds with positive first Chern class. See the survey paper [PhS] for the current status about this problem and also the paper [Wan] in this volume for more detailed discussion of this problem.

It might be worthwhile to note that in spite a lot of works on Kähler-Einstein metrics on Kähler manifolds with positive first Chern class, it seems that these Kähler-Einstein metrics have not been used too much to understand structures of Kähler manifolds with positive first Chern class as in the case when the first Chern class is nonpositive, for example, the Calabi-Yau manifolds, for example, as in the paper [77-1] by Yau, or as in the list in §5.

The booklet [86-8] (or [87-7]) also contains many other problems, questions and comments on them, besides an overview of the status of geometric analysis up to that point.

Another problem in [92-11] is the following:

Problem 48 [92-11]. *Let M be a complete manifold with nonnegative Ricci curvature. Is the space of harmonic functions with polynomial growth finite dimensional when the growth rate is fixed? Does $\mathbb{R}^n$ have the maximal dimension of polynomial growth harmonic functions? Since the author's announcement of this problem at IMU lecture in 1981, Li and Tam [LiT1] have done the best work on this problem. When M is Kähler, we want to know whether the algebra of polynomial growth holomorphic functions is finitely generated or not. The generators give rise to a holomorphic map of M into $\mathbb{C}^n$. What do the image and fiber look like? One can replace functions by holomorphic sections of line bundles with bounded curvature.*

This problem 48 in [92-11] has been solved and led to a lot of work. See [CoM] [LiW] [Li] and references there.

There are several questions on rigidity of locally symmetric spaces from points of views different from the usual Mostow and Margulis rigidity results:

Problem 12 [92-11]. *.... Let M be a compact manifold with nonnegative curvature whose integral cohomology ring is the same as the cohomology ring of an irreducible symmetric space with rank greater than one. Is M symmetric?*

Problem 81 [92-11]. Let M_1 and M_2 be two complete Kähler manifold with bounded curvature and finite volume. Suppose M_1 and M_2 are (properly) homotopic to each other. Can one find a homotopy equivalence between M_1 and M_2 with bounded energy. This is important in studying the rigidity problems [86-5].

Another type of rigidity is discussed in

Problem 86 [92-11]. *Given two locally irreducible Hermitian symmetric space of noncompact type M_1 and M_2. If the moduli spaces of flat vector bundles over M_1 and M_2 are isomorphic to each other for each fiber dimension, is M_1 isomorphic to M_2?*

The problem 14 is concerned with metric spaces:

Problem 14 [92-11]. *When does a compact simplicial complex admit a metric with nonpositive curvature? This is an interesting question even when the dimension is equal to two. Can most theorems related to the fundamental group of manifolds with nonpositive curvature be generalized to these spaces? For example, can one define the rank of such spaces? Is there s useful concept of locally symmetric spaces? Can one use topological date to characterize metric complex that is covered by Barhat-Tits building? .... Is the fundamental group of a compact space with negative curvature "rigid" in some sense? For example, is the mapping class group of the fundamental group finite?...*

Another unusual problem is

Problem 21 [92-11]. *Given a surface in $\mathbb{R}^3$, e.g., the face of a human being, what is the most efficient geometric way to describe the "features" of the surface. ...*

Yau's work on computer vision is related to this problem. See [Gu] fora summary of his work in related areas.

There are many other problems in [92-11]. The problem 47 on the Martin boundary of the universal cover of a rank 1 compact manifold with nonpositive curvature is still completely open. The difficulty is in the case of those manifolds whose sectional curvature is not strictly negative. In fact, the geometry of such rank one manifolds whose sectional curvature is not strictly negatively is still not well-understood.[3]

The list in [00-17] highlights 10 open problems in geometry and supplemented the list in [93-6]. These 10 problems are the same as the 10 problems near the end of [00-16]. The difference is that there are fairly detailed comments on the problems in [00-17]. They are concerned with isometric embedding; the spectrum of the Laplacian of a complete manifold, for example, conditions for a set of discrete numbers in $\mathbb{R}^+$ to be realized as the spectrum; explicit methods to exhibit a large class of minimal submanifolds in $\mathbb{R}^n$ or S^n; classification of compact Einstein manifolds; classification of Riemannian manifolds with special holonomy groups and prove that there are only finite number of deformation types of manifolds with each given special holonomy group; integrable complex structure on manifolds with almost complex structure of dimensions bigger than or equal to 6; conditions for a complex manifold to be Kähler and every Kähler manifold can be deformed to an algebraic manifold; stable holomorphic structure on complex vector bundles over algebraic manifolds; structure of the singular sets of an elliptic variational problem; and full and rigorous geometric explanation of the mirror symmetry for Calabi-Yau manifolds.

As mentioned in the above paragraph, there are also 10 open problems at the end of the paper [00-13].

11 Books written and co-written by Yau

Yau has written and co-written the following books:

1. [08-15] X. Gu, S.T. Yau, *Computational conformal geometry*, Advanced Lectures in Mathematics (ALM), 3. International Press, Somerville, MA; Higher Education Press, Beijing, 2008. vi+295 pp.

2. [98-4]S. Salaff, S.T. Yau, *Ordinary differential equations*, Second edition. International Press, Cambridge, MA, 1998. vi+72 pp.

3. [97-14] R.Schoen, S.T. Yau, *Lectures on harmonic maps*, Conference Proceedings and Lecture Notes in Geometry and Topology, II. International Press, Cambridge, MA, 1997. vi+394 pp.

4. [94-8] R. Schoen, S.T. Yau, *Lectures on differential geometry*, Conference Proceedings and Lecture Notes in Geometry and Topology, I. International Press, Cambridge, MA, 1994. v+235 pp.

5. [86-8] S.T. Yau, *Nonlinear analysis in geometry*, Monographies de L'Enseignement Mathématique 33. Série des Conférences de l'Union Mathématique Internationale 8. L'Enseignement Mathématique, Geneva, 1986. 54 pp.

[3]This problem completely changed my mathematical life after I first read it in the fall 1991 when I arrived at MIT as a Moore instructor and was looking for a problem with temporary office mate Seth Stafford. Since I could not prove it, I tried to work on the Martin compactification of symmetric spaces of noncompact type with rank at least 2 whose sectional curvature is nonpositive but not strictly negative. Then this led to other problems on compactifications of symmetric spaces and locally symmetric spaces.

The booklet [86-8] (or [87-7]) is the extended version of three lectures given by Yau at ETH-Zürich in 1981, giving a broad overview of the role of these equations in geometry. It also emphasized connections of geometry with physics, topology and algebraic geometry. It provided within fifty pages a breathtaking panoramic view of the ever expanding and broad field of geometric analysis.

It started out with a classification of some problems into three types: local problems, semi-local problems, and global problems. For example, local isometric embedding problems are local problems, and understanding singularities in nonlinear hyperbolic and elliptic systems is a semi-local problem. According to Yau [87-7], global problems roughly study analytic structures over compact spaces. For noncompact spaces, the structures are required to be complete in some way. For geometric objects defined by these structures, the goal is to understand their evolution for all time and their asymptotic behaviors. There are two kinds of basic problems:

1. Given a complete analytic structure, how does one deduce global information from local data? This is a uniqueness question.

2. Given the topology of a space, can we put certain analytic structures over this space? This is an existence question.

The rest of this booklet consists of six sections. Besides summaries of known results, it also contains many open problems and comments on them. The headings of the six sections and some subsections are as follows:

§1. Eigenvalues and harmonic functions.

§2. Yamabe's equation and conformally flat manifolds.

§3. Harmonic maps: 1. Existence, uniqueness and regularity. 2. Noncompact manifolds. 3. Rigidity. 4. Harmonic maps in physics.

§4. Minimal submanifolds.

§5. Kähler geometry: 1. Complex and almost complex structures. 2. Kähler and algebraic structures. 3. Uniformization: A. Elliptic manifolds. B. Parabolic manifolds. C. Hyperbolic manifolds. 4. Analytic objects: A. Holomorphic maps and vector bundles. B. Analytic cycles.

§6. Canonical metrics over complex manifolds: 1. The Bergman, Kobayashi-Royden and Caratheodory metrics. 2. Kähler-Einstein metrics on compact Kähler manifolds. 3. Hermitian manifolds and stable vector bundles. 4. Chern number inequalities. 5. Kähler-Einstein metrics on noncompact manifolds. 6. Ricci flat metrics on noncompact manifolds.

The book with Schoen [94-8] was based on lecture series given by Schoen and Yau at Princeton University in 1983 and at the University of California, San Diego, in 1984 and 1985. The book contains significant results in differential geometry and global analysis, many of which were then new results due to Schoen and Yau. For example, constructions of harmonic functions on and the Martin compactifications of simply connected negatively Riemannian manifolds, bounds on eigenvalues and the heat kernels are some of the highlights. It emphasizes differential equations on Riemannian manifolds and the relation between curvature and topology of Riemannian manifolds.

Specifically, the topics covered in this books are as follows:

1. Comparison theorems in Riemannian geometry,
2. Gradient estimates for harmonic functions,
3. Martin boundary and the geodesic boundary of a complete, simply connected manifold of bounded negative sectional curvature,
4. Bounds on eigenvalues and eigenfunctions, and the Sobolev inequality and the isoperimetric inequality,
5. Gradient estimates and Harnack inequality for heat kernels, upper bounds for heat kernels,
6. The Yamabe problem and conformal deformation of scalar curvatures,
7. Structure and invariant of conformally flat manifolds.

There is also a Chinese translation of this book [94-8], and the second edition of this was published in December 2007.

The book with Schoen [97-14] was based on lectures given by Schoen and Yau in 1985 on harmonic maps. It is not an elementary textbook, but rather a collection of some of the most important topics on harmonic maps and their applications.

The headings of the chapters as as follows:

1. Harmonic maps for surfaces,
2. Compactifications of Teichmüller space,
3. Harmonic maps of Kähler manifolds with constant negative holomorphic sectional curvatures,
4. Minimal surfaces in a Kähler surface,
5. Stable minimal surfaces in Euclidean space,
6. The existence of minimal immersions of 2-spheres,
7. Manifolds with positive curvature on totally isotropic two-planes,
8. Compact Kähler manifolds of positive bisectional curvature,
9. Analytic aspects of the harmonic map problem,
10. Sobolev spaces and harmonic maps for metric space targets,
11. Moduli spaces of harmonic maps, compact group actions and the topology of manifolds with non-positive curvature,
12. Harmonic maps and the topology of stable hypersurfaces and manifolds with non-negative Ricci curvature,
13. Harmonic maps and superrigidity.

A Chinese translation of this book [97-14] was published near the end of 2007.

The book [98-4] with Salaff was a text book on ordinary differential equations and written when Yau was a second year college student.

12 Books edited and co-edited by Yau

Yau has edited and co-editted many books. We will put them in three groups: (1) books in the well-known series *Current developments in Mathematics*, which are proceedings of the annual conference with the same title, co-organized by Yau, (2) books in the well-known series *Surveys in differential geometry*, as supplements to *Journal of Differential Geometry*, (3) and other books not in the above series.

We list them in the reversed order by starting with books not in the two series.

1. [08-13]K.S. Lau, Z.P. Xin, S.T. Yau, *Third International Congress of Chinese Mathematicians*, Part 1, 2, AMS/IP Studies in Advanced Mathematics, 42, pt. 1, 2. American Mathematical Society, Providence, RI; International Press, Somerville, MA, 2008. Part 1: lxx+432 pp.; Part 2: pp. i–lxx and 433–874.

2. [08-12]L. Ji, K. Liu, L. Yang, S.T. Yau, *Geometry, analysis and topology of discrete groups*, Advanced Lectures in Mathematics (ALM), 6. International Press, Somerville, MA; Higher Education Press, Beijing, 2008. viii+468 pp.

3. [08-9] K. Liu, S.T. Yau, C. Zhu, *Superstring theory*, Advanced Lectures in Mathematics (ALM), 1. International Press, Somerville, MA; Higher Education Press, Beijing, 2008. ii+348 pp.

4. [06-2] N. Yui, S.T. Yau, J.D. Lewis, *Mirror symmetry. V*, AMS/IP Studies in Advanced Mathematics, 38. American Mathematical Society; International Press, 2006. x+576 pp.

5. [06-4] L.Ji, J.S.Li, H.W.Xu, S.T.Yau, *Lie groups and automorphic forms*, AMS/IP Studies in Advanced Mathematics, 37. American Mathematical Society; International Press, 2006. x+239 pp.

04-9 04-9C. S. Lin, L.Yang, S.T. Yau, *Second International Congress of Chinese Mathematicians*, Proceedings of the congress (ICCM2001) held in Taipei, December 17–22, 2001. New Studies in Advanced Mathematics, 4. International Press, Somerville, MA, 2004. xlvi+687 pp.

6. [03-22] H.Cao, B. Chow, S.C. Chu, S.T. Yau, *Collected papers on Ricci flow*, Series in Geometry and Topology, vol. 37. International Press, 2003. viii+539 pp.

7. [03-19] S.T.Yau, *The founders of index theory: reminiscences of Atiyah, Bott, Hirzebruch, and Singer*, International Press, 2003. liv+358 pp. (A more polished version was published in April 2005).

8. [03-6] S.Y.Chang, C.S. Lin, H.T.Yau, *Lectures on partial differential equations*, Proceedings in honor of Louis Nirenberg's 75th birthday, New Studies in Advanced Mathematics, 2. International Press, 2003. vi+238 pp.

9. [02-11] E. D'Hoker, D.Phong, S.T.Yau, *Mirror symmetry. IV*, AMS/IP Studies in Advanced Mathematics, 33. American Mathematical Society; International Press, 2002. x+381 pp.

10. [02-15] Shuxing Chen, S.T.Yau, *Geometry and nonlinear partial differential equations*, AMS/IP Studies in Advanced Mathematics, 29. American Mathematical Society; International Press, 2002. x+237 pp.

11. [01-10] C.Vafa, S.T.Yau, *Winter School on Mirror Symmetry, Vector Bundles and Lagrangian Submanifolds*, AMS/IP Studies in Advanced Mathematics, 23. American Mathematical Society; International Press, 2001. x+377 pp.

12. [01-13] L.Yang, S.T.Yau, *First International Congress of Chinese Mathematicians*, AMS/IP Studies in Advanced Mathematics, 20. American Mathematical Society; International Press, 2001. lxxvi+518 pp.

13. [99-20] D.Phong, L.Vinet, S.T.Yau, *Mirror symmetry. III*, AMS/IP Studies in Advanced Mathematics, 10. American Mathematical Society; International Press, 1999. x+312 pp.

14. [98-9] (expanded version of [92-12]) S.T.Yau, *S. S. Chern. A great geometer of the twentieth century*, Expanded edition, International Press, 1998.

15. [98-10] (a revised version of [92-9]) S.T.Yau, *Mirror symmetry. I*, Revised reprint of Essays on mirror manifolds [Internat. Press, Hong Kong, 1992]. AMS/IP Studies in Advanced Mathematics, 9. American Mathematical Society; International Press, 1998. xiv+444 pp.

16. [97-9] J.Coates, S.T.Yau, *Elliptic curves, modular forms & Fermat's last theorem*, Second edition. International Press, 1997. iv+340 pp.

17. [97-13] S.T.Yau, *Tsing Hua lectures on geometry & analysis*, International Press, 1997. iv+322 pp.

18. [97-17] B.Greene, S.T. Yau, *Mirror symmetry. II*, AMS/IP Studies in Advanced Mathematics, 1. American Mathematical Society; International Press, 1997. xvi+844 pp.

19. [95-4] S.T.Yau, *Geometry, topology, & physics*, International Press, 1995. iv+538 pp.

20. [95-6] C.S.Liu, S.T.Yau, *Chen Ning Yang. A great physicist of the twentieth century*, International Press, 1995. viii+465 pp.

21. [94-9] R.Penner, S.T.Yau, *Perspectives in mathematical physics*, International Press, 1994. iii+307 pp.

22. [93-10] R.Greene, S.T.Yau, *Differential geometry: Riemannian geometry*, Proceedings of Symposia in Pure Mathematics, 54, Part 3. American Mathematical Society, 1993. xxii+710 pp.

23. [93-11] R.Greene, S.T.Yau, *Differential geometry: partial differential equations on manifolds*, Proceedings of Symposia in Pure Mathematics, 54, Part 1. American Mathematical Society, 1993. xxii+560 pp.

24. [93-12] R.Greene, S.T.Yau, *Differential geometry: geometry in mathematical physics and related topics*, Proceedings of Symposia in Pure Mathematics, 54, Part 2. American Mathematical Society, 1993. xxii+655 pp.

25. [92-9] S.T.Yau, *Essays on mirror manifolds*, International Press, 1992. vi+502 pp.

26. [92-12] S.T.Yau, *Chern—a great geometer of the twentieth century*, International Press, 1992, vi+320.

27. [87-10] S.T.Yau, *Mathematical aspects of string theory*, World Scientific Publishing Co., 1987. x+654 pp.

28. [82-11] S.T.Yau, *Seminar on Differential Geometry*, Annals of Mathematics Studies, 102. Princeton University Press, 1982. ix+706 pp.

We will make only a few comments. The book [82-11] is the first book edited by Yau. It contained all the papers, except those in areas of closed geodesics and minimal surfaces, which were presented in the seminars of the special program in differential geometry, with particular emphasis on partial differential equations, at the Institute for Advanced Study, Princeton, 1979-1980.

It consists of survey papers, for example, the long survey by Yau [82-7] titled *Survey on partial differential equations in differential geometry* and the paper [ChGM] titled *L^2-cohomology and intersection homology of singular algebraic varieties*, and original papers such as [82-8]. Another important feature of this book is the problem section at the end by Yau, which contained over 100 problems [82-9] as mentioned above. This book, in particular, the problem section by Yau, has had a great impact on the development on geometric analysis.

The book [87-10] edited by Yau is the proceedings of the conference held at the University of California, San Diego, California, July 21–August 1, 1986. This conference is one of the first in the currently very active field of mathematical physics, in particular interaction between mathematics and string theory. The book includes papers from diverse branches of mathematics which arise naturally in string theory, and many papers are high quality research papers in final form, for example, the paper [Ti86] by Tian on smoothness of the universal deformation space of compact Calabi-Yau manifolds.

The impact of this book on the development on string theory, in particular, mirror symmetry is enormous. This can be seen through several other books on mirror symmetry edited Yau later.

The book [92-12] is a collection of papers dedicated to S. S. Chern on the occasion of his 79th birthday. It contains articles about both the life and major mathematical achievements of Chern. It also contains an updated version of the list of open problems of Yau [82-9] first published in the book [82-11]. This book was later expanded in [98-9].

A further expanded Chinese version was published in 2005, as the first volume of a book series *Mathematics and Mathematical People.*

The book [92-9] titled *Essays on mirror manifolds* is a first result of combined efforts of both sides (physicists and mathematicians) to understand the phenomenon of mirror symmetry between Calabi-Yau manifolds. Until now this has been a one-way street, the physicists producing the ideas and results and mathematicians trying to put them on solid mathematical ground. To help overcome problems of language and methods, which differ greatly between algebraic geometry and theoretical physics, a workshop on mirror manifolds was held at the MSRI at Berkeley, May 6–8, 1991, and this book can, in some sense, be thought of as the proceedings of the workshop.

The book [97-17] co-edited with Greene is titled *Mirror symmetry. II.* It is the first sequel to the very successful [92-9] *Essays on mirror manifolds*, a revised version of it was reissued as [98-10] with a new title *Mirror symmetry. I.*.

Other sequels with the common title *Mirror symmetry* are *Mirror symmetry III* [99-20], *Mirror symmetry IV* [02-11] *Mirror symmetry V* [06-2].

The book [01-10] co-edited with Vafa is titled *Winter School on Mirror Symmetry, Vector Bundles and Lagrangian Submanifolds* and is the Proceedings of the school held at Harvard University, Cambridge, MA, January 1999.

The book [03-19] is titled *The founders of index theory: reminiscences of Atiyah, Bott, Hirzebruch, and Singer*. It consists of some articles about the life and work of these four people and related topics such as seminars and math research institutes.

Besides the above books, Yau has also edited many volumes in two important series: *Surveys in differential geometry* and *Current developments in Mathematics*. Since these books contain substantial survey papers on topics in the mainstream mathematics, we list the titles of papers in each volume, hoping that they also shed an interesting light on modern mathematics.

The books in the series *Surveys in differential geometry* edited or co-edited by Yau include the following volumes:

1. Volume 1 [91-10]. Proceedings of the Conference on Geometry and Topology held at Harvard University, Cambridge, Massachusetts, April 27–29, 1990. It contains the following articles:

   (a) R.Bott, *Stable bundles revisited*,

   (b) G. D'Ambra and M. Gromov, *Lectures on transformation groups: geometry and dynamics*,

   (c) J.Kollár, *Flips, flops, minimal models, etc.*,

   (d) R.Schoen, *A report on some recent progress on nonlinear problems in geometry*,

   (e) E.Witten, *Two-dimensional gravity and intersection theory on moduli space.*

2. Volume 2 [95-3]. Proceedings of the Conference on Geometry and Topology held at Harvard University, Cambridge, Massachusetts, April 23–25, 1993. It contains the following papers:

   (a) M.Atiyah, *Reflections on geometry and physics*,

   (b) R.Hamilton, *The formation of singularities in the Ricci flow*,

   (c) B.Lawson, *Spaces of algebraic cycles*,

   (d) Y.Manin, *Problems on rational points and rational curves on algebraic varieties*,

   (e) L.Simon, *Rectifiability of the singular sets of multiplicity* 1 *minimal surfaces and energy minimizing maps*,

   (f) C.Taubes, *Homology cobordism and the simplest perturbative Chern-Simons* 3*-manifold invariant*,

   (g) C.Taubes, *Metabolic cobordisms and the simplest perturbative Chern-Simons* 3*-manifold invariant.*

3. Volume 3 [98-8]. *Lectures on geometry and topology in honor of the 80th birthday of Chuan-Chih Hsiung*, a conference held at Harvard University, Cambridge, MA, May 3–5, 1996. It contains the following papers:

   (a) J.Bismut, *Local index theory, eta invariants and holomorphic torsion: a survey*,

   (b) J.P.Otal, *Thurston's hyperbolization of Haken manifolds*,

   (c) D.Gabai, *Quasi-minimal semi-Euclidean laminations in* 3*-manifolds*,

   (d) P.Kronheimer, *Embedded surfaces and gauge theory in three and four dimensions*,

   (e) C.Taubes, *The geometry of the Seiberg-Witten invariants.*

4. Volume 5 [99-18]. *Differential geometry inspired by string theory.* It contains the following papers:

    (a) P.Aspinwall, $K3$ *surfaces and string duality*,

    (b) J.Bismut, F.Labourie, *Symplectic geometry and the Verlinde formulas*,

    (c) J.Bryan, N.Leung, *Counting curves on irrational surfaces*,

    (d) M.Gross, *Special Lagrangian fibrations. II. Geometry. A survey of techniques in the study of special Lagrangian fibrations*,

    (e) B.Lian, K.Liu, S.T.Yau, *Mirror principle. I*,

    (f) B.Lian, K.Liu, S.T.Yau, *Mirror principle. II*,

    (g) B.Lian, S.T.Yau, *Differential equations from mirror symmetry*,

    (h) K.Liu, *Heat kernels, symplectic geometry, moduli spaces and finite groups*,

    (i) J.Li, G.Tian, *A brief tour of GW invariants.*

5. Volume 7 [00-12]. *Papers dedicated to Atiyah, Bott, Hirzebruch, and Singer.* It contains the following papers:

    (a) M.Atiyah, *The geometry of classical particles*,

    (b) E.Brieskorn, *Singularities in the work of Friedrich Hirzebruch*,

    (c) C.Ciliberto, G.van der Geer, *The moduli space of abelian varieties and the singularities of the theta divisor*,

    (d) R.Cohen, E.Lupercio, G.Segal, *Holomorphic spheres in loop groups and Bott periodicity*,

    (e) S.Donaldson, *Moment maps and diffeomorphisms*,

    (f) D.Freed, *Dirac charge quantization and generalized differential cohomology*,

    (g) D.Goldfeld, S.Zhang, *The holomorphic kernel of the Rankin-Selberg convolution*,

    (h) V.Guillemin, C.Zara, *Equivariant de Rham theory and graphs*,

    (i) R.Harvey, B.Lawson, *Morse theory and Stokes' theorem*,

    (j) F.Hirzebruch, *The Atiyah-Bott-Singer fixed point theorem and number theory*,

    (k) N.Hitchin, *The moduli space of complex Lagrangian submanifolds*,

    (l) R.Kadison, *Which Singer is that?*

    (m) P.Li, *Curvature and function theory on Riemannian manifolds*,

    (n) B.Lian, K.Liu, S.T.Yau, *Mirror principle. III*,

    (o) B.Lian, K.Liu, S.T.Yau, *Mirror principle. IV*,

    (p) Y.Manin, *Three constructions of Frobenius manifolds: a comparative study*,

    (q) R.Penrose, *On Ricci-flat twistor theory*,

    (r) W.Schmid, K.Vilonen, *On the geometry of nilpotent orbits*,

    (s) C.Taubes, *Seiberg-Witten invariants, self-dual harmonic 2-forms and the Hofer-Wysocki-Zehnder formalism*,

    (t) C.Vafa, *Unifying themes in topological field theories*,

(u) E.Witten, *Noncommutative Yang-Mills theory and string theory.*

6. Volume 8 [03-8]. *Lectures on geometry and topology held in honor of E. Calabi, H. Blaine Lawson, Y. T. Siu and Karen Uhlenbeck*, It contains the following papers:

   (a) M.Atiyah, J.Berndt, *Projective planes, Severi varieties and spheres*,

   (b) J.Cheeger, *Degeneration of Einstein metrics and metrics with special holonomy*,

   (c) H.Colding, C.De Lellis, *The min-max construction of minimal surfaces*,

   (d) C.Croke, M.Katz, *Universal volume bounds in Riemannian manifolds*,

   (e) J.P.Demailly, T.Peternell, *A Kawamata-Viehweg vanishing theorem on compact Kähler manifolds*,

   (f) S.Donaldson, *Moment maps in differential geometry*,

   (g) D.Fisher, G.Margulis, *Local rigidity for cocycles*,

   (h) C.LeBrun, *Einstein metrics, four-manifolds, and differential topology*,

   (i) N.Leung, *Topological quantum field theory for Calabi-Yau threefolds and G_2-manifolds*,

   (j) W.Meeks, *Geometric results in classical minimal surface theory*,

   (k) A.Nahmod, *On global existence of wave maps with critical regularity*,

   (l) E.Viehweg, K.Zuo, *Discreteness of minimal models of Kodaira dimension zero and subvarieties of moduli stacks*,

   (m) S.Wolpert, *Geometry of the Weil-Petersson completion of Teichmüller space.*

7. Volume 9 [04-8]. *Eigenvalues of Laplacians and other geometric operators.* It contains the following papers.

   (a) M.Barlow, *Anomalous diffusion and stability of Harnack inequalities*,

   (b) G.Besson, *From isoperimetric inequalities to heat kernels via symmetrisation*,

   (c) F.Chung, *Discrete isoperimetric inequalities*,

   (d) T.Colding, W.Minicozzi, *An excursion into geometric analysis*,

   (e) A.Grigoryan, Y.Netrusov, S.T.Yau, *Eigenvalues of elliptic operators and geometric applications*,

   (f) M.Ledoux, *Spectral gap, logarithmic Sobolev constant, and geometric bounds*,

   (g) L.Lovász, *Discrete analytic functions: an exposition*,

   (h) W.Meeks, J.Pérez, *Conformal properties in classical minimal surface theory*,

   (i) R.Neel, D.Stroock, *Analysis of the cut locus via the heat kernel*,

   (j) L.Saloff-Coste, *Analysis on Riemannian co-compact covers*,

   (k) F.Shahidi, *Functoriality and small eigenvalues of Laplacian on Riemann surfaces*,

   (l) S.Zelditch, *The inverse spectral problem.*

8. Volume 10 [06-11] *Essays in geometry in memory of S. S. Chern* contains the following papers.

   (a) B. Dai, C.L.Terng, K. Uhlenbeck, On the space-time monopole equation,

(b) V. Guillemin, S. Sternberg, J. Weitsman, The Ehrhart function for symbols,

(c) K. Liu, Recent results on the moduli space of Riemann surfaces,

(d) William H. Meeks, III, Applications of minimal surfaces to the topology of three-manifolds,

(e) V. Moncrief, An integral equation for spacetime curvature in general relativity,

(f) A. Neitzke, C. Vafa, Topological strings and their physical applications,

(g) R. P. Thomas, Notes on GIT and symplectic reduction for bundles and varieties,

(h) S.T. Yau, Perspectives on geometric analysis,

(i) S.W. Zhang, Distributions in algebraic dynamics.

9. Volume 12 [08-8] *Geometric flows* contains the following papers.

(a) S. Brendle, On the conformal scalar curvature equation and related problems,

(b) A. Chau, L.F. Tam, A survey on the Khler-Ricci flow and Yau's uniformization conjecture,

(c) H.D.Cao, B. Chen, X.P. Zhu, Recent developments on Hamilton's Ricci flow,

(d) C. Gerhardt, Curvature flows in semi-Riemannian manifolds,

(e) J. Krieger, Global regularity and singularity development for wave maps,

(f) V. Moncrief, Relativistic, Teichmüller theory—a Hamilton-Jacobi approach to 2 + 1-dimensional Einstein gravity,

(g) Lei Ni, Monotonicity and Li-Yau-Hamilton inequalities,

(h) C. Sinestrari, Singularities of mean curvature flow and flow with surgeries,

(i) M.T. Wang, Some recent developments in Lagrangian mean curvature flows.

10. Voulme 13 [09-9] *Geometry, analysis, and algebraic geometry: forty years of the Journal of Differential Geometry* contains the following papers.

(a) D. Auroux, Special Lagrangian fibrations, wall-crossing, and mirror symmetry,

(b) S. Brendle, R. Schoen, Sphere theorems in geometry,

(c) R. Donagi, T. Pantev, Geometric Langlands and non-abelian Hodge theory,

(d) K. Grove, Developments around positive sectional curvature,

(e) C. LeBrun, Einstein metrics, four-manifolds, and conformally Kähler geometry ,

(f) F. Hang, F. Lin, Y. Yang, Existence of Faddeev knots in general Hopf dimensions,

(g) F. Bogomolov, Y. Tschinkel, Milnor K_2 and field homomorphisms,

(h) E. Viehweg, Arakelov inequalities ,

(i) S.T. Yau, A survey of Calabi-Yau manifolds.

11. Voulme 14 [09-12] *Geometry of Riemann surfaces and their moduli spaces* contains the following papers.

(a) E. Arbarello, M. Cornalba, Divisors in the moduli spaces of curves,

(b) R. L. Cohen, Stability phenomena in the topology of moduli spaces,
(c) G. Farkas, Birational aspects of the geometry of $\mathcal{M}_g$,
(d) S. Grushevsky, I. Krichever, The universal Whitham hierarchy and the geometry of the moduli space of pointed Riemann surfaces,
(e) J. Harris, Brill-Noether theory,
(f) P. Hubert, E. Lanneau, M. Möller, $GL_2^+(R)$-orbit closures via topological splittings,
(g) J. Jost , S. T. Yau, Harmonic mappings and moduli spaces of Riemann surfaces,
(h) Y. P. Lee, R. Vakil, Algebraic structures on the topology of moduli spaces of curves and maps,
(i) K. Liu, X. Sun, S.T. Yau, Recent development on the geometry of the Teichmller and moduli spaces of Riemann surfaces,
(j) V. Markovic, D. Saric, The universal properties of Teichmller spaces,
(k) H. Masur, Geometry of Teichmller space with the Teichmüller metric,
(l) I. Morrison, GIT constructions of moduli spaces of stable curves and maps,
(m) Y. Ruan, Riemann surfaces, integrable hierarchies, and singularity theory.

The books in the series *Current developments in Mathematics* edited or co-edited by Yau include the following volumes:

1. Volume 1 [94-5]. Papers from the seminar held in Cambridge, MA, April 1995. It contains the following papers.
   (a) H.Darmon, F.Diamond, R.Taylor, *Fermat's last theorem*,
   (b) M.Lyubich, *Renormalization ideas in conformal dynamics*,
   (c) I.Madsen, *Algebraic K-theory and traces*,
   (d) C.McMullen, *The classification of conformal dynamical systems*,
   (e) G.Tian, *Quantum cohomology and its associativity*,
2. Volume 2 [97-8]. Papers from the seminar held in Cambridge, MA, 1996. It contains the following papers.
   (a) R.Borcherds, *Automorphic forms and Lie algebras*,
   (b) G.Heckman, E.Opdam, *Harmonic analysis for affine Hecke algebras*,
   (c) E.Hrushovski, *Stability and its uses*,
   (d) Y.Meyer, *Wavelets, paraproducts, and Navier-Stokes equations.*
3. Volume 3 [99-19]. Papers from the seminar held in Cambridge, MA, 1997. It contains the following papers.
   (a) A.Connes, *Trace formula on the adele class space and Weil positivity*,
   (b) L.Evans, *Partial differential equations and Monge-Kantorovich mass transfer*,
   (c) P.Sarnak, *Quantum chaos, symmetry and zeta functions. Lecture I. Quantum chaos*,

(d) P.Sarnak, *Quantum chaos, symmetry and zeta functions. Lecture II. Zeta functions*,
(e) W.Soergel, *Character formulas for tilting modules over quantum groups at roots of one*,
(f) A.Suslin, *Voevodsky's proof of the Milnor conjecture*,
(g) U.Frisch, *Is there finite-time blow-up in 3-D Euler flow?*
(h) E.Lieb, *Problems*,
(i) B.Mazur, *Open problems in number theory*,
(j) C.Taubes, *Open problems in geometry and topology*,
(k) J.Bourgain, *The Cauchy problem for nonlinear Schrödinger equation (NLS) with critical nonlinearity*,
(l) W.Craig, *Singularities of Schrödinger's equation and recurrent bicharacteristic flow*,
(m) G.David, *Global minimizers of the Mumford-Shah functional*,
(n) L.Escauriaza, *The modified Bers conjecture*,
(o) M.Grillakis, *The wave map problem*,
(p) N.Nadirashvili, *Geometry of nodal sets and multiplicity of eigenvalues*,
(q) T.Wolff, *Questions related to the dimension of Kakeya sets*,
(r) Z.Xin, *Open problems in partial differential equations arising from fluid dynamics.*

4. Volume 4 [99-17] contains the following papers.

(a) B.Greene, *Mirror symmetry: a brief review of the first 10 years*,
(b) B.Lian, K.Liu, S.T.Yau, *Mirror principle, a survey*,
(c) P.Kronheimer, *Developments in symplectic topology*,
(d) L.van den Dries, *o-minimal structures and real analytic geometry*,
(e) H.T.Yau, *Asymptotic solutions to dynamics of many-body systems and classical continuum equations*,
(f) L.S.Young, *Geometric and ergodic theory of hyperbolic dynamical systems.*

5. Volume 5 [99-13] contains the following papers.

(a) H.Bray, R.Schoen, *Recent proofs of the Riemannian Penrose conjecture*,
(b) J.Conway, C.Goodman-Strauss, N.Sloane, *Recent progress in sphere packing*,
(c) G.Henniart, *A report on the proof of the Langlands conjectures for* $\mathrm{GL}(N)$ *over* p*-adic fields*,
(d) G.Laumon, *The Langlands correspondence for function fields following Laurent Lafforgue.*

6. Volume 6 [01-9] contains the following papers.

(a) M.Broué, *Reflection groups, braid groups, Hecke algebras, finite reductive groups*,
(b) W.E, *Stochastic hydrodynamics*,
(c) W.Gowers, *Arithmetic progressions in sparse sets*,

(d) J.Kollár, *The topology of real algebraic varieties*,

(e) O.Schramm, *Scaling limits of random processes and the outer boundary of planar Brownian motion.*

7. Volume 7 [02-8] contains the following papers.

   (a) G.Carlsson, *Recent developments in algebraic K-theory*,

   (b) D.Freed, *K-theory in quantum field theory*,

   (c) E.Lieb, J.Yngvason, *The mathematical structure of the second law of thermodynamics*,

   (d) E.Lieb, R.Seiringer, J.Solovej, J.Yngvason, *The ground state of the Bose gas*,

   (e) S.Zhang, *Elliptic curves, L-functions, and CM-points.*

8. Volume 8 [03-4] contains the following papers.

   (a) A.Bressan, *One dimensional hyperbolic systems of conservation laws*,

   (b) M.Haiman, *Combinatorics, symmetric functions, and Hilbert schemes*,

   (c) R.Hain, *Periods of limit mixed Hodge structures*,

   (d) S.Kudla, *Modular forms and arithmetic geometry*,

   (e) Y.Minsky, *End invariants and the classification of hyperbolic 3-manifolds*,

   (f) L.Saper, *On the cohomology of locally symmetric spaces and of their compactifications.*

9. Volume 9 [03-21] contains the following papers.

   (a) S.Fomin, A.Zelevinsky, *Cluster algebras: notes for the CDM-03 conference*,

   (b) H.Iwaniec, *Automorphic number theory*,

   (c) C.Voisin, *On some problems of Kobayashi and Lang; algebraic approaches.*

10. Volume 10 [06-10] contains the following papers.

    (a) H. Hofer, A general Fredholm theory and applications,

    (b) L. Caffarelli, A homogenization method for non variational problems,

    (c) W. H. Meeks, III, Applications of minimal surfaces to the topology of three-manifolds,

    (d) E. Lindenstrauss, Adelic dynamics and arithmetic quantum unique ergodicity

    (e) C. Skinner, Main conjectures and modular forms

11. Volume 11 [07-3] contains the following papers.

    (a) E. D'Hoker, D. H. Phong, Complex geometry and supergeometry,

    (b) D. Sullivan, String topology background and present state,

    (c) E.H. Lieb, The stability of matter and quantum electrodynamics,

    (d) J. Y. Cai, Holographic algorithms,

    (e) S. DeBacker, The fundamental lemma: what is it and what do we know?

(f) P.W. Shor, The additivity conjecture in quantum information theory,

(g) M. Kisin, Modularity of 2-dimensional Galois representations.

12. Volume 12 [08-11] contains the following papers.

    (a) L. Clozel, The Sato-Tate conjecture,

    (b) S. Gukov, E. Witten, Gauge theory, ramification, and the geometric Langlands program,

    (c) D. Li, Y.G. Sinai, Complex singularities of the Burgers system and renormalization group method,

    (d) P. Seidel, A biased view of symplectic cohomology,

    (e) T. Tao, Global behaviour of nonlinear dispersive and wave equations.

13. Volume 13 [09-7] contains the following papers.

    (a) B. Green, Three topics in additive prime number theory,

    (b) Y. Kawamata, Finite generation of a canonical ring,

    (c) J. Li, Recent progress in GW-invariants of Calabi-Yau threefolds,

    (d) D. H. Phong, Jacob Sturm, Lectures on stability and constant scalar curvature,

    (e) Y.T. Siu, Techniques for the analytic proof of the finite generation of the canonical ring,

    (f) C. Taubes, Notes on the Seiberg-Witten equations, the Weinstein conjecture and embedded contact homology.

14. Volume 14 [09-11] contains the following papers;

    (a) M. Dafermos, The Evolution Problem in General Relativity,

    (b) L. Lovász, Very Large Graphs,

    (c) J. Lurie, On the Classification of Topological Field Theories,

    (d) W. H. Meeks III, and J. Pérez, Properly embedded minimal planar domains with infinite topology are Riemann minimal examples,

    (e) K. Ono, Unearthing the Visions of a Master: Harmonic Maass Forms and Number Theory.

13 Ph.D. students of Yau

Yau has been a very effective teacher. His student seminar has certainly be an intergral part of training students. His broad interests are reflected in talks of this seminar. The following is the list of his Ph.D. students graduated up to the summer of 2009.

1. John Nash, Stanford University, 1976,

2. Richard Schoen, Stanford University, 1977, schoen@math.stanford.edu, currently at Stanford University.

3. Thomas Parker, Stanford University, 1980, parker@math.msu.edu, currently at Michigan State University.

4. Andrejs Treibergs, Stanford University, 1980, treiberg@math.utah.edu, currently at University of Utah.

5. Richard Klotz, Stanford University, 1982,

6. Shigetoshi Bando, Princeton Univ, 1983, bando@math.tohoku.ac.jp, currently at Tohoku University, Japan.

7. Robert Bartnik, Princeton University, 1983, robert.bartnik@sci.monash.edu.au, currently at Monash University, Australia.

8. Leslie Saper, Princeton University, 1984, saper@math.duke.edu, currently at Duke University.

9. Mark Stern, Princeton Univ, 1984, stern@math.duke.edu, currently at Duke University.

10. Huai-Dong Cao, Princeton University, 1986, huc2@lehigh.edu, currently at Lehigh University.

11. Bennett Chow, Princeton University, 1986, benchow@math.ucsd.edu, currently at UC San Diego.

12. Alex Freire, Princeton University, 1988, freire@math.utk.edu, currently at University of Tennessee.

13. Gang Tian, Harvard University, 1988, tian@Math.Princeton.EDU, currently at Princeton University.

14. Jun Li, Harvard University, 1989, jli@math.stanford.edu, currently at Standford University.

15. Tzuemn-Renn Lin, University of California, San Diego, 1989.

16. Wen-Xiang Wang, Princeton University, 1989, wwang@csusb.edu, currently at California State University, San Bernardino.

17. Fangyang Zheng, Harvard University, 1989, zheng@math.ohio-state.edu currently at Ohio State University.

18. Rui-Tao Dong, University of California, San Diego, 1990, rdong@trestlecorp.com

19. Y. W. Sung, Brandeis University, 1990

20. Steven Lu, Harvard University, 1990, lu@math.uqam.ca, currently at Université du Québec á Montréal.

21. Wan-Xiong Shi, Harvard University, 1990.

22. Lizhen Ji, Northeastern University, 1991, lji@umich.edu, currently at University of Michigan.

23. S.Y. Kao, Brandeis University, 1991, sjkao@math.nthu.edu.tw, currently at National Tsinghua University, Taiwan.

24. Ping-Zen Ong, University of California, San Diego, 1991, ong@math.ntu.edu.tw, currently at National Taiwan University.

25. Nai-Chung Leung, Massachusetts Institute of Technology, 1993, leung@ims.cuhk.edu.hk, currently at Chinese University of Hong Kong.

26. Kefeng Liu, Harvard University, 1993, liu@math.ucla.edu, currently at UCLA.

27. Wei-Dong Ruan, Harvard University, 1995, ruan@math.kaist.ac.kr, (ruan@math.uic.edu).

28. Tian-Jun Li, Brandeis University, 1996, TJLI@MATH.UMN.EDU, currently at University of Minnesota.

29. Ai-Ko Liu, Harvard University, 1996, currently at UBS at Stamford, Connecticut.

30. Xiao Zhang, Chinese University of Hong Kong, 1996, xzhang@math08.math.ac.cn, currently at Institute of Mathematics, Academy of Mathematics and Systems Sciences, Academia Sinica, Beijing.

31. Huazhang Luo, MIT, 1998, lhz777@gmail.com, currently at Lehman Brothers in London.

32. Chin-Lung Wang, Harvard University, 1998, dragon@math.ncu.edu.tw, dragon@math.cts.nthu.edu.tw, currently at National Central University, Taiwan.

33. Mu-Tao Wang, Harvard University, 1998, mtwang@cpw.math.columbia.edu, currently at Columiba University.

34. Charles Doran, Harvard University, 1999, doran@math.ualberta.ca, currently at University of Alberta.

35. Shu-Yu Hsu, Chinese University of Hong Kong, 1999, syhsu@math.ccu.edu.tw, currently at National Chung Cheng University.

36. John Loftin, Harvard University, 1999, loftin@andromeda.rutgers.edu currently at University of Rutgers.

37. Christine Taylor, Harvard University, 2000.

38. Mao-Pei Tsui, Brandeis University, 2000, Mao-Pei.Tsui@Utoledo.edu, currently at University of Toledo.

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14 Partial list of papers and books of Yau

To the best of our knowledge, we have put down most of the published papers and books of Yau according to MathSciNet. Since he has written so many and some papers are not even listed in MathSciNet, some might have slipped through.

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