Math 431 Exam 1 Review Sheet

7 October 2008

- Chapter 1. Euclid's geometry. We did not spend much time directly discussing the results of this section. The primitives "point", "line", and "incident with" (or "lies on") are discussed in Chapter 2. The remaining primitives are discussed in Chapter 3, but, so far, we have considered only "between".
- Chapter 2. Logic and incidence geometry.
  - Construct structured proofs using axioms and rules of logic. Do *not* memorise the numbering or naming of results (a list will be provided), but be familiar with their statements and uses.
  - Particularly, understand the rules of negation (the negation of an 'or' statement is an 'and' statement, and conversely; the negation of a 'for some' statement is a 'for all' statement, and conversely).
  - Be able to prove the propositions of incidence geometry. Do *not* memorise the proofs; instead, be able to re-create them "from scratch".
  - Specify and understand interpretations of axioms. These are sometimes point-and-line (or -curve) drawings, sometimes "plain English" descriptions, and sometimes formal descriptions in another axiomatic language (including that of set theory).
  - Verify that an interpretation is, or is not, a model for a theory by checking whether the axioms of the theory are true for the interpretation. Understand the relationship between provability and models: A statement that fails in some model is not provable in the theory; but a statement that is true in some (or even every) model might still not be provable in the theory.
  - Know some models, and some interpretations that are not models, of incidence geometry.
  - Understand the idea of duality. Be able to reason about dual geometries, and to write down the duals of geometric statements. Know when the dual of a model is, or is not, another model.
  - Parallelism properties.
    - An affine plane is a model of incidence geometry with "exactly the right number" of parallel lines. Affine planes can be obtained by deleting a line from a projective plane.
    - A projective plane is a model of incidence geometry with no parallel lines (in which all lines have at least 3 points). Projective planes can be obtained by adding a line "at ∞" to an affine plane.
    - Hyperbolic planes are models of incidence geometry with too many parallel lines. So far, we have only seen one, rather small example.
    - What effect does the number of parallel lines have on the geometry? What results are true for affine planes that are not true for projective planes, and *vice versa*?
- Chapter 3. Hilbert's axioms.
  - Understand the definitions of segments and rays in terms of betweenness; particularly, understand how the definition captures the idea of

direction, and of "stopping". Understand, informally, why these definitions require the new primitive of "between", rather than just the primitives of incidence geometry.

- Understand how the 4 axioms of betweenness (attempt to) capture our intuition about the shape and behaviour of lines. Particularly, notice that they prohibit some of the examples of incidence geometries discussed in Chapter 2. Give examples of interpretations that satisfy some, but not all, of the axioms of betweenness.
- Be able to prove Propositions 3.1–3.4, and the corollaries that appear among them. Note that the proofs of these propositions require results from the exercises.
- Understand and reason with the concept of a "side" of the plane, or of a line. Be able to answer the questions: On which "side" is the dividing line? How many "sides" are there, and why?