Math 431 Exam 2 Review Sheet

November 10, 2008

A natural question is "Is Exam 2 cumulative?"; or, at greater length, "Will Exam 2 include questions about the material on Exam 1?" The answer is necessarily yes. In all mathematics, but particularly in geometry, what comes later builds on what comes before, so that it is essentially impossible to ask questions now that do not involve our previous concepts. (For example, our current axiom system includes all the axioms of incidence geometry.) Nonetheless, the emphasis for this exam will be much more heavily on new than on old material.

As always, this handout is meant only as a rough guide to the most important ideas from the past month. All topics that we covered in class can appear on the exam, even if they do not appear here.

We have spent the time since the first exam discussing Chapter 3. The second exam will consider only material up to p. 129; in particular, you need not study the continuity axioms in the remainder of Chapter 3. Although material from Chapter 4 will not be directly included, you may find it interesting additional practice with the material from Chapter 3 to read the beginning of that chapter.

Since we can prove more interesting things with our richer axiom system than we could with the axiom system that was available for Exam 1, the exam will involve more proofs and fewer questions about models and interpretations. All proofs on the exam can be organised in any convenient fashion (structured, paragraph-style, or a mix), as long as I can understand it. It is fair game for me to ask you to prove a result that we have already done in class or on the homework, so you should be able to prove such results; but such "repeats" will not constitute a significant part of the exam.

It is extremely good practice to try to sketch out an argument before you present it formally, and to draw pictures beforehand and along the way. Although this will never earn you full credit in the absence of a complete proof, it can make the difference between no credit and substantial partial credit (if you've captured important ideas.)

Chapter 3. Hilbert's axioms.

- Betweenness.
  - Betweenness statements, by definition, can only be made for triples of points. Use Proposition 3.3 and its corollary to chain betweenness statements involving more than 3 points.
  - Know why we can decompose segments into subsegments, rays into subrays, and so forth by using the axioms of betweenness. See Propositions 3.4 and 3.5, for example.
  - Use the trick of proving that something interesting happens on a ray by proving that it happens on the associated line, and doesn't happen on the opposite ray. See Exercise 3.12, for example.
  - Know how betweenness for *rays* behaves in comparison to betwenness for *points*. See particularly Exercise 3.14.
  - Define the interior of an angle and prove things about it. Proposition 3.8 is a good source of statements about the interior of an angle, but *the proof of Proposition 3.8(c) is difficult.* A proof of similar difficulty on the exam would be broken into smaller parts, with hints.

  - Understand the behaviour of lines and rays with respect to triangles. See Pasch's theorem and Exercise 3.12, for example.
- Congruence.
  - Even with the list of axioms in front of you, it will be much harder to reason with the congruence axioms if you work directly from your statements than if you have some informal versions of the axioms in mind. For example, think *first* that axiom C1 involves moving segments; use this informal idea to decide whether it is the appropriate axiom for your problem; and only then, if it is appropriate, worry about the exact combination of letters to use in your application of the axiom. Don't underestimate naming conventions. Using primes (for example, A') to convey some idea of relationship or similarity (i.e., "this has something to do with A") can be a valuable organisational guide.
  - As a particular case of the above, recognise how much information is necessary to move a segment, angle, or triangle. Many other problems can usefully be thought of as moving a certain arrangement. For example, Proposition 3.12 says that we can move a segment with marked point.
  - Understand why segment and angle addition and subtraction are so called. For example, what does Proposition 3.11 have to do with subtraction? These problems can sometimes be fruitfully addressed by actually physically writing out and manipulating "equations" as an informal guide to how a formal proof should go.
  - Be able to add information to a new congruence problem to turn it into a situation where we already understand congruence. See, for example, the proof of Proposition 3.17.
  - Use existing congruence information for angles and segments to prove congruence of triangles, and then use existing congruences of triangles to prove new congruences of angles and segments.
  - Understand how the congruence tests that we know follow from the SAS congruence tests, and how to use them to prove the validity of other congruence tests.