Math 431 Exam 3 Review Sheet

December 4, 2008

The final exam for this course will be cumulative. In particular, the contents of the Exam 1 and Exam 2 review sheets should be regarded as part of this review sheet. The emphasis for the final exam will be slightly more on new material than on old material, but you should still be sure to review some older concepts (such as the idea of models, and, in particular, of non-Euclidean models of incidence geometry) that we covered early on and have not much revisited.

As always, this handout is meant only as a rough guide to the most important ideas from the past month. All topics that we covered in class can appear on the exam, even if they do not appear here.

The second exam covered almost all of the material that we consider from Chapter 3. Since then, we have concentrated almost entirely on Chapter 4, with only a brief excursion back into the continuity principles discussed in Chapter 3 on pp. 129–131. We did *not* discuss the measurement axioms, namely, Archimedes's axiom and its successors, that appear beginning on p. 132, and these will not be considered on the exam. We covered the part of Chapter 4 prior to "Saccheri and Lambert quadrilaterals" (i.e., up to p. 176), except that we omitted Measurement Theorem 4.3 on pp. 169–170. Nonetheless, we discussed the results appearing on pp. 171–172 by using circumlocutions that avoid having to talk about measures at all.

The exam will continue to be mostly proof-oriented. However, constructing examples and counter-examples, and providing precise versions of statements (for instance, converting "the greater side is opposite the greater angle" into the text of Proposition 4.5) are very valuable abilities. Once again, it is extremely good practice to try to sketch out an argument before you present it formally, and to draw pictures beforehand and along the way. Although this will never earn you full credit in the absence of a complete proof, it can make the difference between no credit and substantial partial credit (if you've captured important ideas).

Chapter 3. Hilbert's axioms.

- Continuity properties. We spent very little time discussing the continuity properties, and used them only in one of our constructions of midpoints of segments. Don't worry about them much, but do make sure that you understand the uses of them that we made in class.

Chapter 4. Neutral geometry.

- The alternate interior angles theorem (Theorem 4.1). Note that, without the assumption of a parallel postulate, we only have a theorem that allows us to identify parallel lines, given congruent angles; *not* one that allows us to identify congruent angles, given parallel lines.
- Theorem 4.1 allows us to *prove* that parallel lines exist, in relative abundance. Understand how this proof proceeds, and why this is *not* the same as proving the Euclidean parallel postulate.
- Theorem 4.1 gives some restriction on the angles of a triangle: There cannot be 2 distinct non-acute angles.
- The exterior angle theorem (Theorem 4.2).

  - The *proof* of this theorem provides an interesting application of Theorem 4.1—it finds a contradiction by showing that 2 lines constructed to have a point in common must be parallel.
  - This theorem is most interesting for its *applications*. It captures many of our important ideas about why a point must fall in a certain position on a triangle. See, for example, the proof of the angle-angle-side test for congruence (Proposition 4.1).
- Theorem 4.2 gives more restrictions on the angles of a triangle: "Any 2 of them must sum to less than a straight angle". Be able to explain what this means *without* using the language of measures of angles (as on p. 171).
- Our familiar congruence tests for triangles are SAS (Axiom C6), ASA (Proposition 3.17), SSS (Proposition 3.22), and AAS (Proposition 4.1). Be able to prove all of these (by reductions to previous ones) and to recognise when it is appropriate to apply them.
- A less familiar congruence test is the "hypotenuse–leg criterion" (Proposition 4.2), one of the few cases when SSA works. Extra problem 2 on Homework 12 originally claimed that the SSA never worked (except for right triangles), but this is not quite true. Understand why SSA usually fails, and when it succeeds.
- We proved Proposition 3.19 (angle addition) using Axiom C3 (segment addition); Proposition 3.20 (angle subtraction) using Proposition 3.11 (segment subtraction); and Proposition 3.21 (ordering of angles) using Proposition 3.13 (ordering of segments). Be able to reduce statements about angles to statements about segments, when appropriate. Where is this done in Chapter 4?
- Be able to prove and apply the intuitively obvious statements Propositions 4.5 and 4.6, and "triangle inequality" (on p. 171), about sidelengths of triangles and their relations to angles. For example, note that Proposition 4.6 proves that one segment is longer than another by placing both inside a triangle and computing an angle. See Problem 5 on Homework 12 (Exercise 4.9 from the text) for details.
- Prove and use the implications of the Euclidean parallel postulate. See pp. 173–176.
  - The more familiar form of the alternate interior angles theorem (the one that deduces that lines are parallel by knowing that angles are congruent).
  - Transitivity of parallelism (if 2 distinct lines are each parallel to a 3rd, then they are parallel to one another)—see Exercise 4.10. This can be used to prove statements about perpendicularity and parallelism (Propositions 4.9 and 4.10).
  - The sum of the measures of the angles in a triangle is $180°$. Be able to state this without using the language of measures of angles.