My AIM Faculty Portrait

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Q: What is a wave? A: A basic phenomenon of continuum mechanics.

- A disturbance propagating with some well-defined speed.
- The disturbance might not change its form as it propagates (traveling wave).
- The disturbance might change its form in a predictable way as it propagates (e.g. a “breather”, or a “galloping front”).
- Otherwise hard to define precisely... but we all know intuitively what waves are.
From fundamental laws of physics to tractable mathematical models.

- Nondimensionalization and identification of key dimensionless parameters.
- Asymptotic Reduction: assume one of the parameters is small and expand.

Analysis of mathematical models (e.g. the wave equation $u_{tt} = u_{xx}$).

- Special solutions. Symmetries.
- Numerical methods and scientific computation.
In most physical systems, waves that are big (compared to ...) are necessarily understood by means of nonlinear models (PDE, etc.), e.g. the Korteweg-de Vries equation $u_t + uu_x + u_{xxx} = 0$.

- This makes their mathematical theory complicated — no superposition principle.
- It also makes possible robust coherent structures:
The SIAG on NWCS

One of the 19 Activity Groups within SIAM (SIAGs) is the SIAG on Nonlinear Waves and Coherent Structures (NWCS). It organizes a SIAM conference every other year. Frequent topics include:

- Wave patterns and pattern formation.
- Dynamical stability of nonlinear waves. (Linearization leads to $du/dt = Au$.)
- Applications of nonlinear wave theory (including biology and neuroscience, materials science, optics and photonics, water waves including tsunamis and rogue waves).
- Soliton theory and Integrable Systems.
Soliton Theory

It turns out that an amazing nexus occurs within nonlinear wave theory:

- The same basic nonlinear equations (e.g. Korteweg-de Vries) show up again and again from diverse applications (universality).
- Many of these basic nonlinear equations are **integrable systems**:
  - Having numerous exact solutions, e.g. **solitons**.
  - Having remarkable stability properties allowing for “nonlinear superpositions” of solitons.
  - Having numerous remarkable symmetries (connecting solutions).
  - Having an associated transform technique, an **inverse-scattering transform**. This is a nonlinear analogue of the Fourier transform for solving general initial-value problems. First discovered in the late 1960’s.

Much of my research involves using mathematical tools of soliton theory to extract physically interesting information about nonlinear waves.
An Example of a Current Research Project

With AIM student Alfredo Wetzel, we study the Benjamin-Ono equation:

\[ u_t + uu_x + \epsilon H[u_{xx}] = 0, \quad \epsilon > 0, \quad H[f](x) := \frac{1}{\pi} \int_{\mathbb{R}} \frac{f(y)}{y - x} dy \]

which models waves \( u = u(x, t) \) on the interface between two fluid layers. An application to keep in mind: the prediction of the depth of contaminants trapped at the subsurface interface. Deepwater Horizon spill, summer 2010.
Our mathematical interest is in the behavior of solutions when the dispersion parameter $\epsilon$ is small:

Alfredo is using an inverse-scattering transform and methods from complex analysis and asymptotic and functional analysis to establish remarkably simple asymptotic formulae for $u(x, t)$ that describe the generated oscillations.
Trains of Morning Glory Clouds
Where could my co-advisor come from?

Some examples:

- Earth Sciences (Oceanography, e.g. Alfredo’s co-advisor Brian Arbic)
- Electrical Engineering and Computer Science (Nonlinear Optics Group)
- Atmospheric, Oceanic, and Space Science
- Physics
- Naval Architecture and Marine Engineering
What kind of math background would help?

Key AIM Core Courses:
- Math 555 (Complex Variables)
- Math 556 ( Applied Functional Analysis)
- Math 557 ( Asymptotic Analysis)
- Math 572 ( Numerical Methods for Differential Equations)

Other courses:
- Math 596 (Complex Analysis)
- Math 602 (Functional Analysis)
- Math 650 (Fourier Analysis)
- Math 651 (When I teach it as a soliton theory course, otherwise a reading course)
- Math 656/657 (Partial Differential Equations)
Any Questions?

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Thank You!