Problem session 4

As usual, all schemes are assumed to be of finite type over an algebraically closed field k.

Problem 1. Let X be an integral scheme, and $\mathcal{F}$ a coherent sheaf on X. Show that there is an open subset $U \subseteq X$ such that $\mathcal{F}|_U$ is locally free.

Problem 2. Show that a scheme X is affine if and only if $X_{\rm red}$ is affine.

Problem 3. Let $f \colon X \to Y$ be an affine morphism of separated schemes. Show that if $\mathcal{F}$ is a quasicoherent sheaf on X, then we have isomorphisms

$$H^i(X, \mathcal{F}) \simeq H^i(Y, f_*(\mathcal{F}))$$

for every $i \geq 0$.

Problem 4. Let $X = \mathbf{A}^2 \smallsetminus \{0\}$. Compute $H^1(X, \mathcal{O}_X)$, and show that it is infinite-dimensional over k.

Problem 5. Show that for every scheme X, there is an isomorphism

$$\mathrm{Pic}(X) \simeq H^1(X, \mathcal{O}_X^*).$$