

In topology, the notions of the fundamental group and the universal cover are inextricably intertwined. In algebraic geometry, the traditional development of the étale fundamental group is somewhat different, reflecting the perceived lack of a good universal cover. However, I will describe how the usual notions from topology carry over directly to the algebraic and arithmetic setting without change, rectifying imperfections in the étale fundamental group.

One key example is the absolute Galois group scheme, which contains more information than the traditional absolute Galois group, in a choice-free manner, and has a rich arithmetic structure. Its geometric fiber is the classical absolute Galois group as a topological group (the profinite topology *is* the Zariski topology, and comes from geometry). I will also discuss the example of abelian varieties and the Tate module. This is joint work with Kirsten Wickelgren.