

The space of smooth genus 0 curves in projective space has a natural smooth compactification: the moduli space of stable maps, which may be seen as the generalization of the classical space of complete conics. It has a beautiful combinatorial structure. In arbitrary genus, no such natural smooth model is expected, as the space satisfies “Murphy’s Law”. In genus 1, however, the situation remains beautiful and combinatorial. I will describe a natural smooth compactification of the space of elliptic curves in projective space. This space is a blow up of the space of stable maps. It can be interpreted as blowing up the most singular locus first, then the next most singular, and so on, but with a twist — these loci are often entire components of the moduli space. I will give a number of applications in enumerative geometry and Gromov-Witten theory. For example, it has been used by Aleksey Zinger to prove physicists’ famous mirror symmetry prediction for genus 1 Gromov-Witten invariants of a quintic threefold. This is joint work with Zinger.