Homework Set 1

Solutions are due Friday, September 21st.

**Problem 1.** Let $Y$ be the closed algebraic subset of $\mathbb{A}^3$ defined by the two polynomials $x^2 - yz$ and $xz - x$. Show that $Y$ is a union of three irreducible components. Describe them and find their prime ideals.

**Problem 2.** Let $X$ be a topological space.

i) Show that if $Y$ is a subset of $X$ (with the induced topology), then $Y$ is irreducible if and only if its closure $\overline{Y}$ is irreducible.

ii) Suppose that $X$ is Noetherian, and that $Y$ is a subset $X$. Show that if $Y = Y_1 \cup \ldots \cup Y_r$ is the irreducible decomposition of $Y$, then $\overline{Y} = \overline{Y_1} \cup \ldots \cup \overline{Y_r}$ is the irreducible decomposition of $\overline{Y}$.

**Problem 3.** Note that we have a natural identification of $\mathbb{A}^2$ with the Cartezian product $\mathbb{A}^1 \times \mathbb{A}^1$. Show that the Zariski topology of $\mathbb{A}^2$ in *not* the product topology on $\mathbb{A}^1 \times \mathbb{A}^1$ of the two Zariski topologies on each $\mathbb{A}^1$.

**Problem 4.** Show that if $f$ is a polynomial in $k[x_1, \ldots, x_n]$, then the corresponding hypersurface $V(f)$ is irreducible if and only if $f$ has no distinct prime factors.