

Errata for "The geometries of 3-manifolds", Bull. London Math. Soc. 15 (1983), 401-487, by Peter Scott

Page 411: The last sentence of para 2 asserts "Certainly, if all angles are less than π , this can be done."

This is false. The correct statement is that if α , β and γ are numbers between 0 and π , then they form the angles of a spherical triangle if and only if $\alpha + \beta + \gamma > \pi$ and $\alpha + \beta < \gamma + \pi$, $\beta + \gamma < \alpha + \pi$, $\alpha + \gamma < \beta + \pi$. The last three conditions must hold for a spherical triangle, as such a triangle has area less than that of each of the lunes which contain it, and a lune with angle α has area 2α .

Page 432, lines -5 and -6: This sentence gives an unconvincing argument to show that the interior of M cannot be covered by $S^2 \times \mathbb{R}$. Here is a more convincing argument. In the previous sentence, it was observed that as ∂M is non-empty, the interior of M is non-compact and so cannot be covered by S^3 . Hence $\pi_1(M)$ must be infinite. Now we claim that if the interior of M is covered by $S^2 \times \mathbb{R}$, it must be compact, which is again a contradiction. To see this, let G denote the subgroup of $\pi_1(M)$ of index at most 2 which does not interchange the two ends of $S^2 \times \mathbb{R}$. Let Σ denote $S^2 \times \{0\}$, and let X denote $S^2 \times [0, \infty)$. As G must be infinite, there is g in G such that $g\Sigma$ is disjoint from Σ . Thus $gX \not\subseteq X$ or $X \not\subseteq gX$. In either case, it follows that g has infinite order and that the compact region between Σ and $g\Sigma$ is a fundamental region for the action of the cyclic group generated by g . Hence the quotient of $S^2 \times \mathbb{R}$ by this cyclic group is compact, so that the interior of M is also compact.

Page 451, lines 27-30: The sentence "In particular, one can show that a finite subgroup of Σ must be cyclic or dihedral or generalized quaternion." is literally correct, but is quite misleading. The point here is that the dihedral case cannot occur since there is only one unit quaternion of order 2. This led to the definitely incorrect statement "Thus we have free actions of cyclic and dihedral groups on S^3 " in the next sentence.

To correct this, the first sentence should be replaced by the sentence "In particular, one can show that a finite subgroup of Σ must be cyclic or generalized quaternion." And the incorrect statement should be corrected by replacing the word "dihedral" by the phrase "generalized quaternion".

Page 460, line 13: It need not be true that “all the leaves of this foliation are compact”.

Page 468, lines -7 to end: There is a bad misprint here in which two sentences were messed up in the printed version. This paragraph (beginning on line -7 of page 468 and ending on line 3 of page 469) should read as follows:

“The isometry group of Nil generated by \widetilde{Nil} and this circle action is 4-dimensional. It follows, as when we discussed \widetilde{SL}_2 , that this group is the identity component of $Isom(Nil)$. As the map $(x, y, z) \rightarrow (x, -y, -z)$ is an isometry of Nil , we see that $Isom(Nil)$ has at least two components. In order to show that there are no more components of $Isom(Nil)$, we argue as for \widetilde{SL}_2 . We need to find a loop l in \mathbb{E}^2 with a horizontal lift into Nil which is not a loop, i.e. the endpoints are distinct. Recall that the metric on \mathbb{R}^3 is given by $ds^2 = dx^2 + dy^2 + (dz - xdy)^2$. It follows that the horizontal plane at (x, y, z) contains the vectors $(1, 0, 0)$ and $(0, 1, x)$. Now it is clear that if l is the boundary of the square in \mathbb{E}^2 with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$ and $(1, 1)$, then a horizontal lift of l into Nil has distinct endpoints.”