Math 462 Homework 5

1: Verify the following system has a non-hyperbolic fixed point at the origin for some value of μ. Sketch three phase portraits with qualitatively different behaviors. $\dot{x}_1 = \mu x_1 - x_2$,$\dot{x}_2 = x_1 + \mu x_2$

2: Consider the forced Van der Pol equation $\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = A \sin \omega t$, where A is the amplitude, or displacement of the wave function and ω is its frequency. Study the problem (both analytically and numerically) for various A and ω. What do you notice happens to the limit cycle?

3: Completely analyze (with phase portraits) the system

$$\frac{dx}{dt} = f(x)[g(x) - y],$$
$$\frac{dy}{dt} = \delta h(x) y$$

where

$$f(x) \equiv \frac{x}{\frac{x^2}{\alpha} + x + 1},$$
$$g(x) \equiv (1 - \frac{x}{\gamma})(\frac{x^2}{\alpha} + x + 1),$$
$$h(x) \equiv \beta f(x) - 1$$

HINT: I suggest studying this in terms of its general functions. Also, there will be 6 different regions of parameter space to consider and only ONE bifurcation diagram.