Lab 5, Math 462 Due Monday 4/5, 2008

Problem 1: Consider the following

$$\frac{d^2x}{dt^2} + \alpha x = \Gamma \cos \omega t$$

The solution to this equation was provided in class. Run simulations for this equation by varying ω for some fixed α and consider conditions when $\omega^2 \to \alpha$. What do you noticing happening to the solution as you get closer to α?

Problem 2: Now introduce damping, using a constant damping coefficient, k. What does the equation become? Vary both k and ω and explain the differences between the solutions in 1 and the solutions you are now seeing. What is different about the solution? What happens as $\omega^2 \to \alpha$?

Problem 3: To introduce the effects of nonlinearity on resonance consider an example of the Duffing equation with no damping and a weak non-linearity.

$$\frac{d^2x}{dt^2} + \Omega^2 x - \epsilon x^3 = \Gamma \cos t$$

In this case we simply let $\omega = 1$. Find the order one solution for the equation and the order ϵ solution. Consider the periodicity condition $x(\epsilon, t + 2\pi) = x(\epsilon, t)$ What does the solution tell you about resonance? Run some simulations for Ω near resonance.

Problem 4: Now consider the full Duffing equation,

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \alpha x - \delta x^3 = \Gamma \cos \omega t$$

Run simulations for varying parameters and explain what you are seeing? Try at least three different cases.