Lab 5, Math 462 April 11, 2008

Now we consider the Lorenz equation. This is the prototype equation for generating chaos. Of interest here is to study how chaos arises from what appears to be a simple set of differential equations. The equations to study are

$$\frac{dx}{dt} = -\sigma x + \sigma y$$
$$\frac{dy}{dt} = rx - y - zx$$
$$\frac{dz}{dt} = -bz + xy$$

1) Find the equilibrium points of this system and show the stability of the zero fixed point.

2) Graph three solution plots, one each for x, y and z separately for the following values: $r = 28$, $\sigma = 10$, and $b = 8/3$. Use the initial conditions $\bar{x} = (0, 1, 0)$.

3) Now plot two dimensional phase portraits for the case in problem 2. Plot x vs y, x vs z and y vs z. What do you notice about these plots.

4) Now play around with the parameters you were given in problem 1 to see what other type of behavior you can generate. Specifically, what critical value of $r = r_c$ can you find so that the strange attractor no longer exists?