

Math 462: Mathematical Modeling

Instructor: Dr. Patrick Nelson

Homework 1: Due Monday 2/11

1: Classify the fixed points of the following systems of equations and state the type of stability for each one.

1a:

$$\begin{aligned}\frac{dx}{dt} &= 2x - 3y \\ \frac{dy}{dt} &= \alpha y - 2x\end{aligned}$$

1b:

$$\begin{aligned}\frac{dx}{dt} &= y^2 - 3x + 2 \\ \frac{dy}{dt} &= x^2 - y^2\end{aligned}$$

1c:

$$\begin{aligned}\frac{dx}{dt} &= -y + x + xy \\ \frac{dy}{dt} &= x - y - y^2\end{aligned}$$

2: Show that the following differential equation exhibits a pitchfork bifurcation. Draw both cases of the bifurcation diagram

$$\frac{dx}{dt} = ax - bx^3$$

3: Draw the complete phase portrait diagram for the following system of equations. Verify all the fixed points and their stability.

$$\begin{aligned}\frac{dx}{dt} &= x(x - 2) \\ \frac{dy}{dt} &= y(-2x - y + 6)\end{aligned}$$