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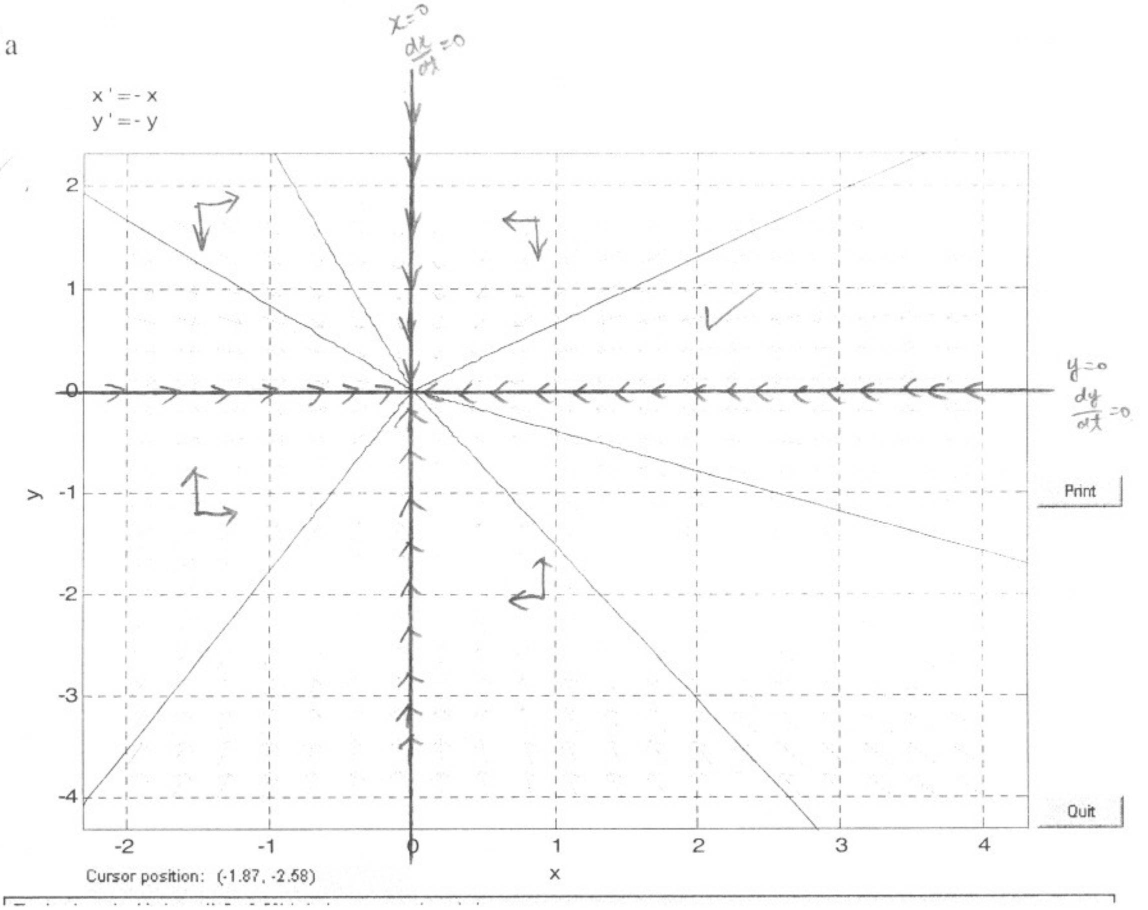
MATH 4?2
LAB #3
84512560
Wei-Ju Tsay

1a



Cursor position: (-1.87, -2.58)

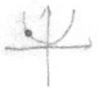
1° fixed point: $(x,y)=(0,0)$

$$\begin{cases}\frac{dx}{dt}=-x=0\\ \frac{dy}{dt}=-y=0\end{cases}$$

2° linearization.

$$\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$\beta=\text{tra}\,J=-2$



$\sigma=\det J=1$

$\beta^2-4\sigma=4-4=0$

∴ stable star #

$\lambda^2+2\lambda+1=0$

$\lambda=-1,-1$ #

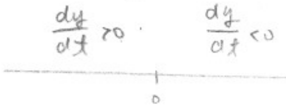
3°. Nullclines $x=0,\ y=0$

$$\begin{cases}\frac{dx}{dt}=0=-x\\ \frac{dy}{dt}=0=-y\end{cases}\ \therefore\ \begin{cases}x=0\\y=0\end{cases}$$

when $x=0$, $\frac{dx}{dt}=0$

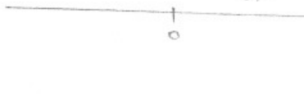
$\frac{dy}{dt}=-y$

✓

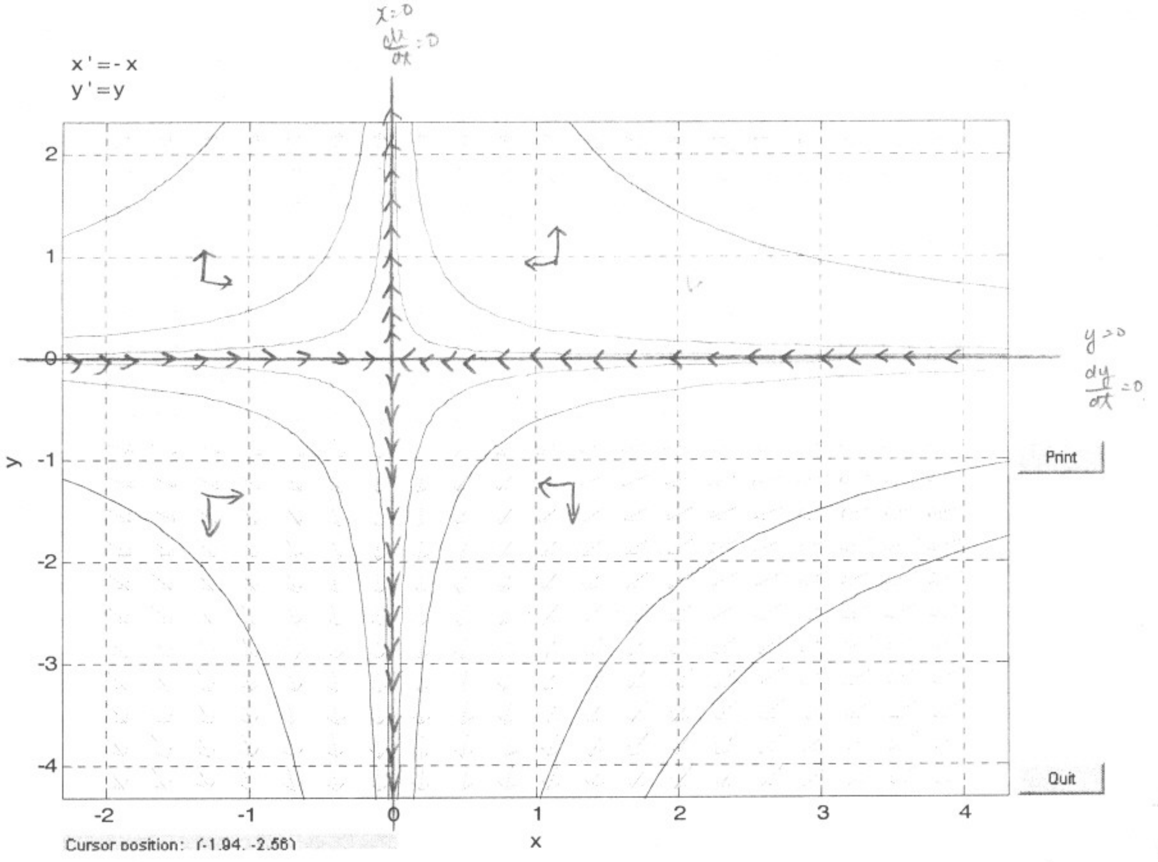


when $y=0$, $\frac{dy}{dt}=0$

$\frac{dx}{dt}=-x$



1c



1° fixed point

$$\begin{cases} \frac{dx}{dt} = -x = 0 \\ \frac{dy}{dt} = y = 0 \end{cases} \qquad (x,y)=(0,0)$$

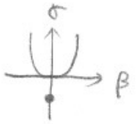
2° linearization

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$J = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\beta = \text{tra} J = 0$

$\gamma = \det J = -1$



∴ SADDLE POINT #

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$ #

3° nullclines : $x=0$, $y=0$

$$\begin{cases} \frac{dx}{dt} = 0 = -x \\ \frac{dy}{dt} = 0 = y \end{cases}$$

when $x=0$, $\frac{dx}{dt}=0$

$\frac{dy}{dt} = y$

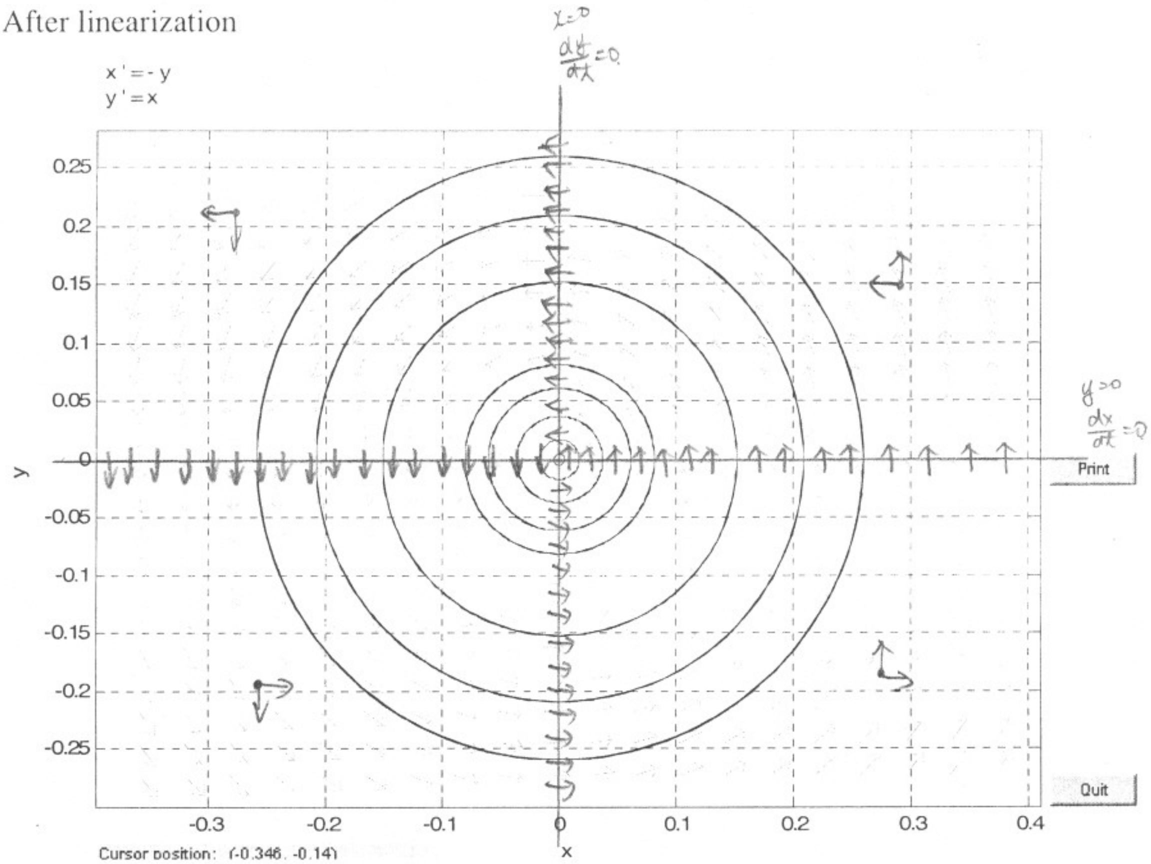
$\frac{dy}{dt} < 0$ 0 $\frac{dy}{dt} > 0$ y

when $y=0$ $\frac{dy}{dt}=0$

$\frac{dx}{dt} = -x$

$\frac{dx}{dt} > 0$ 0 $\frac{dx}{dt} < 0$ x

After linearization



Cursor position: (-0.346, -0.14)

Assume $x_1 = x \quad x_2 = y$

xed point

$$\begin{cases} \frac{dx}{dt} = -y + x(x^2+y^2) = 0 \\ \frac{dy}{dt} = x + y(x^2+y^2) = 0 \end{cases}$$

$$(x^2+y^2) = \frac{y}{x}$$

$$\therefore x + y\frac{y}{x} = 0$$

$$\therefore x^2+y^2 = 0$$

$$\therefore \begin{cases} y=0 \\ x=0 \end{cases}$$

linearization :

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x^2+y^2 & -1+2xy \\ 1+2xy & x^2+3y^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \checkmark$$

$(x,y)=(0,0)$

$$\therefore \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$\beta = \text{tra}J = 0$

$\gamma = \det J = 1$

$\therefore$ center # $\checkmark$

$\lambda^2 + 1 = 0 \qquad \lambda = \pm i$ #

3° nullclines.

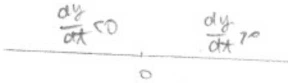
for linearization one.

$\frac{dx}{dt} = -y = 0$

$\frac{dy}{dt} = x = 0$

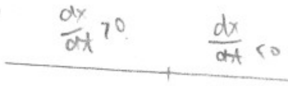
when $y=0$ $\frac{dx}{dt} = 0$

$\frac{dy}{dt} = x$



when $x=0$ $\frac{dy}{dt} = 0$

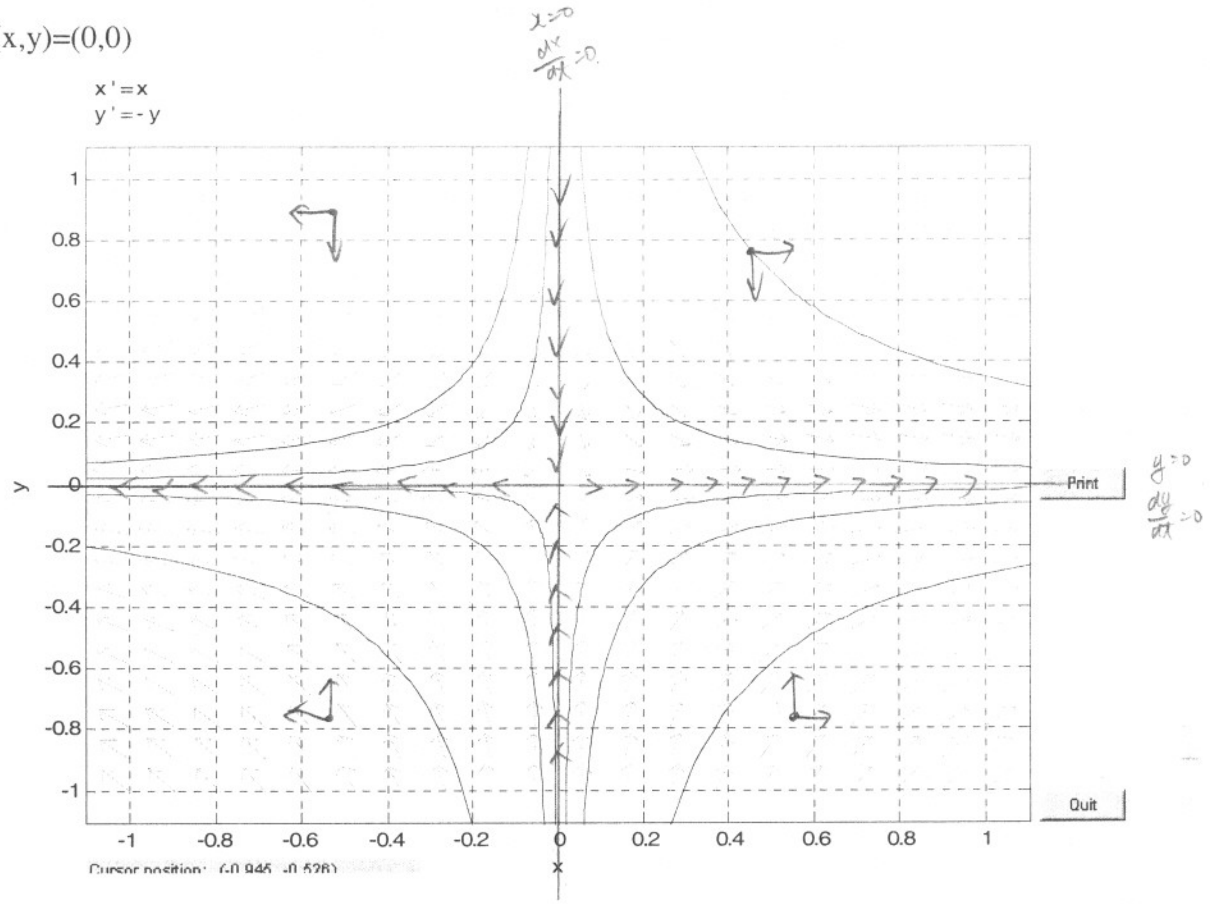
$\frac{dx}{dt} = -y$



original figure = see the first two

After linearization

(x,y)=(0,0)



$(x,y)=(0,0)$

$$\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$J=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ ✓

$\beta = \text{tr} J = 0$

$\gamma = \det = -1$

∴ SADDLE POINT

\#

$x^2-1=0$

$\lambda=\pm 1$

\#

nullclines: $x=0$, y

$\frac{dx}{dt}=x=0$

$\frac{dy}{dt}=-y=0$

when $x=0$ $\frac{dx}{dt}=0$

$\frac{dy}{dt}=-y$

$\frac{dy}{dt}>0$

0

when $y=0$ $\frac{dy}{dt}=0$

$\frac{dx}{dt}=x$

$\frac{dx}{dt}<0$

0

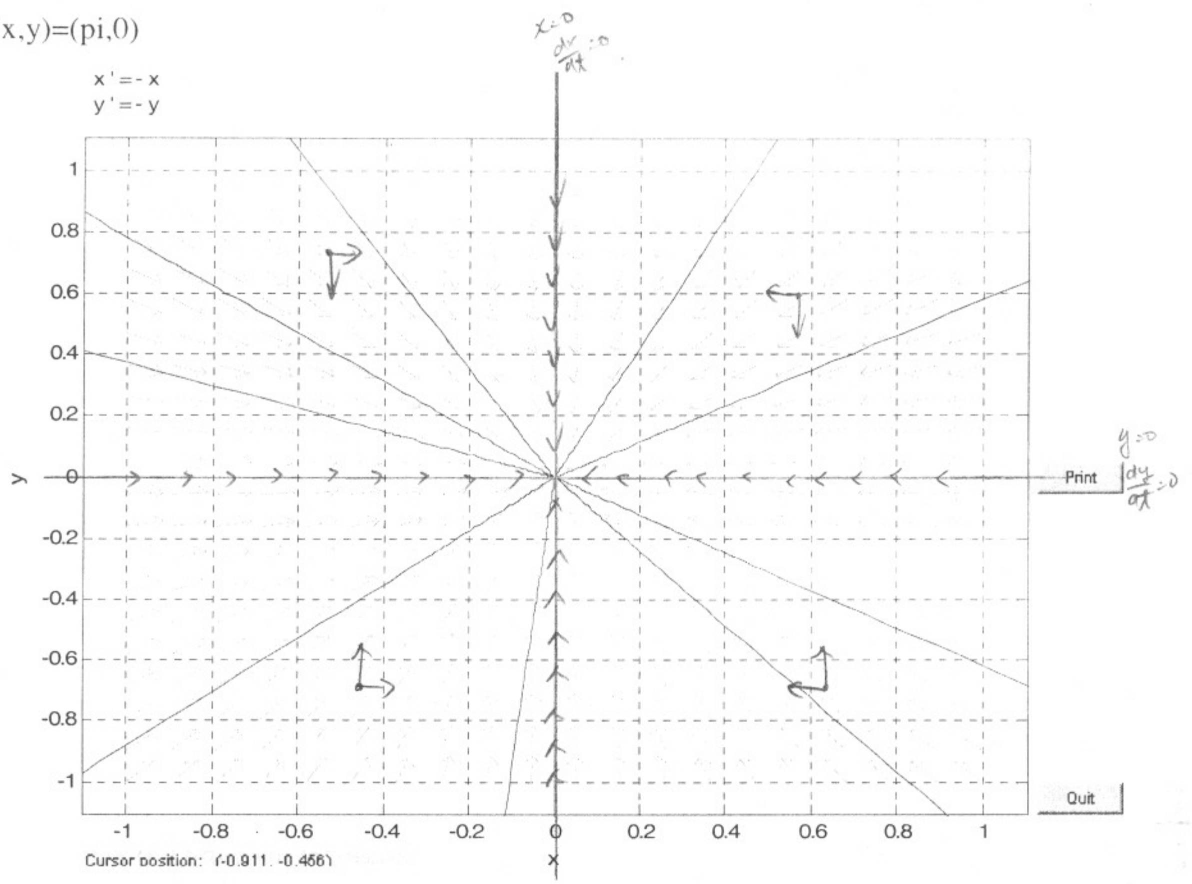
compare with nonlinear case, these p

portrait and flow trajectories are

@ (0,0)

\#

(x,y)=(pi,0)



fixed point

$(x,y) = (\pi, 0)$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$J = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\beta = \text{tra} J = -2$

$\gamma = \det J = 1$

$\beta^2 - 4\gamma = 4 - 4 = 0$

∴ stable star #

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0$

$\lambda = -1, -1$ #

Nullclines: $x=0$, $y=0$

$$\begin{cases} \frac{dx}{dt} = -x = 0 \\ \frac{dy}{dt} = -y = 0 \end{cases}$$

when $x=0$ $\frac{dx}{dt} = 0$

$\frac{dy}{dt} = -y$

$\frac{dy}{dt} > 0$ $\frac{dy}{dt} < 0$

0

when $y=0$ $\frac{dy}{dt} = 0$

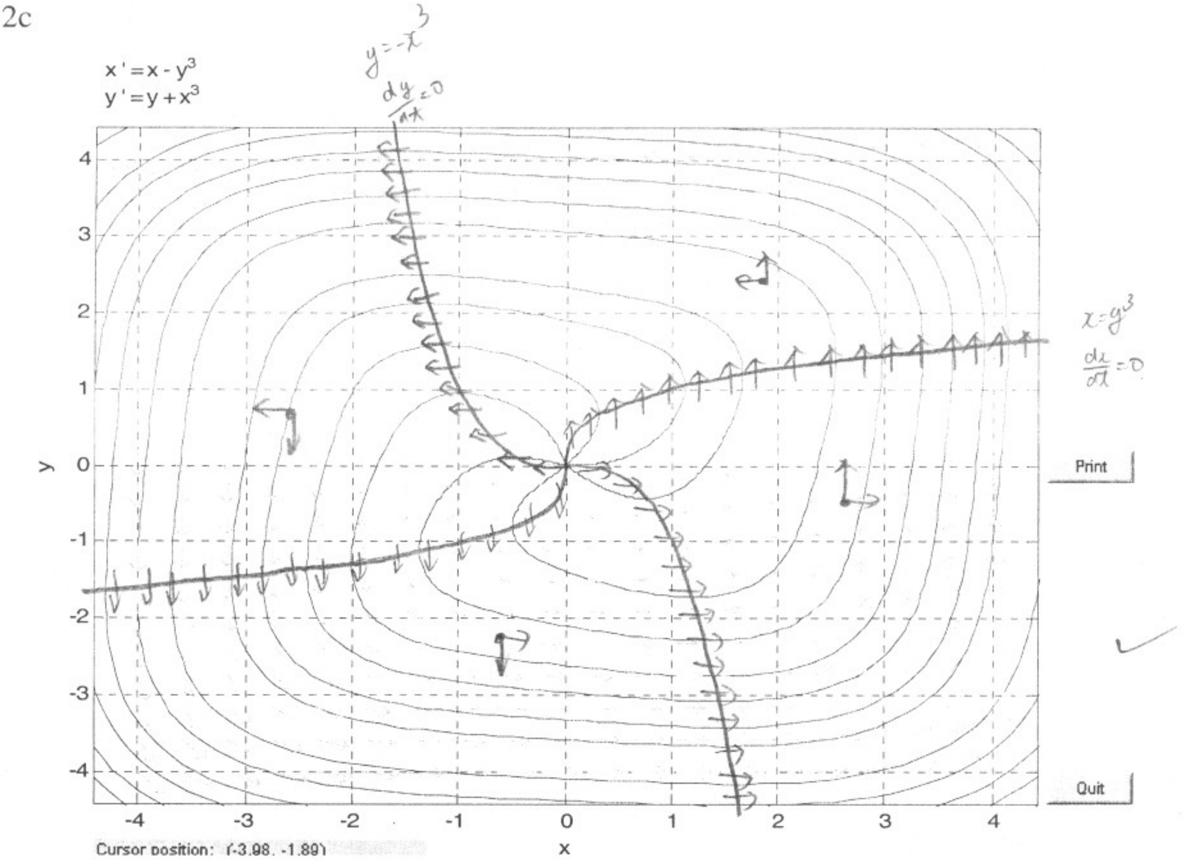
$\frac{dx}{dt} = -x$

$\frac{dx}{dt} > 0$ $\frac{dx}{dt} < 0$

0

In nonlinear phase portrait, that seems to be a stable node instead of stable star @ $(\pi, 0)$ #

2c



Assume $X_1 = x$, $X_2 = y$

fixed point

$$\begin{cases} \frac{dx}{dt} = x - y^3 = 0 & x = y^3 \\ \frac{dy}{dt} = y + x^3 = 0 \end{cases}$$

$y + y^9 = 0$

$y(y^8 + 1) = 0$

$y = 0, \Rightarrow x = 0$

∴ fixed point $(x, y) = (0,0)$

2° linearization

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -3y^2 \\ 3x^2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$\Rightarrow (x, y) = (0, 0)$

$$\therefore \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\beta = \text{tra} J = 2$ $\qquad \beta^2 - 4\sigma = 0$

$\sigma = \det J = 1$

∴ unstable star #

3° Nullclines : $x = y^3$, $y = -x^3$

$$\begin{cases} \frac{dx}{dt} = 0 = x - y^3 \\ \frac{dy}{dt} = 0 = y + x^3 \end{cases}$$

when $x = y^3$. $\frac{dx}{dt} = 0$

$\frac{dy}{dt} = y + x^3 = y + y^9$

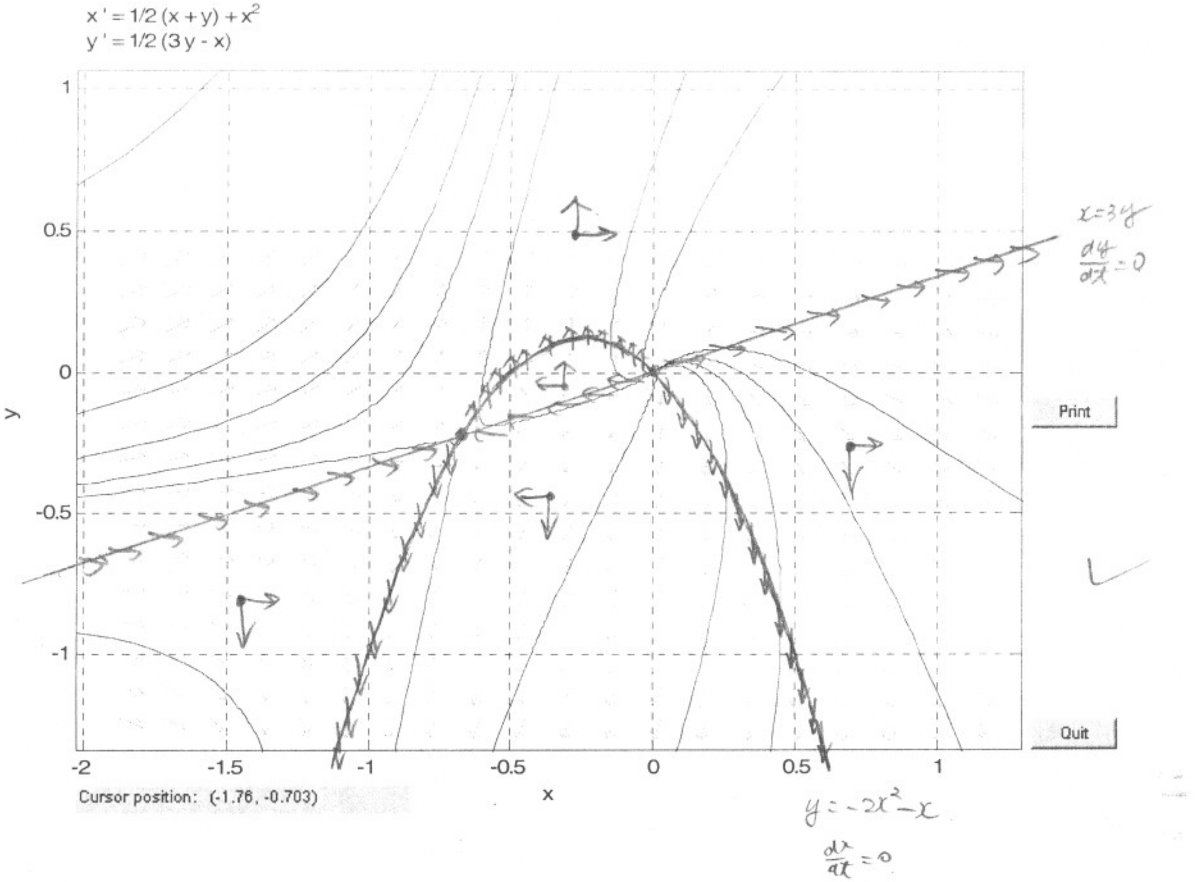
$\frac{dy}{dt} < 0$ $\quad$ 0 $\quad$ $\frac{dy}{dt} > 0$ $\quad$ y

when $y = -x^3$ $\frac{dy}{dt} = 0$

$\frac{dx}{dt} = x + x^9$

$\frac{dx}{dt} < 0$ $\quad$ 0 $\quad$ $\frac{dx}{dt} > 0$ $\quad$ x

From my calculation, it should be unstable star in linear case, but for nonlinear phase portrait, it is a unstable spiral.



$x' = 1/2\,(x + y) + x^2$

$y' = 1/2\,(3\,y - x)$

3° Nullclines.

$$\begin{cases} \frac{dx}{dt} = \frac{1}{2}(x+y)+x^2 = 0 \\ \frac{dy}{dt} = \frac{1}{2}(3y-x) = 0 \end{cases}$$

when $\frac{1}{2}(x+y)+x^2=0$

$(x+y)+2x^2=0$

$y=-2x^2-x$, $\frac{dx}{dt}=0$

∴ $\frac{dy}{dt} = \frac{1}{2}\left(3(-2x^2-x)-x\right)$

$= \frac{1}{2}\left(-6x^2-4x\right)$

$= -3x^2-2x = x(-3x-2)$

$\frac{dy}{dt}<0$. $\frac{dy}{dt}>0$ $\frac{dy}{dt}<0$

$-\frac{2}{3}$ 0 x

when $x=3y$, $\frac{dy}{dt}=0$

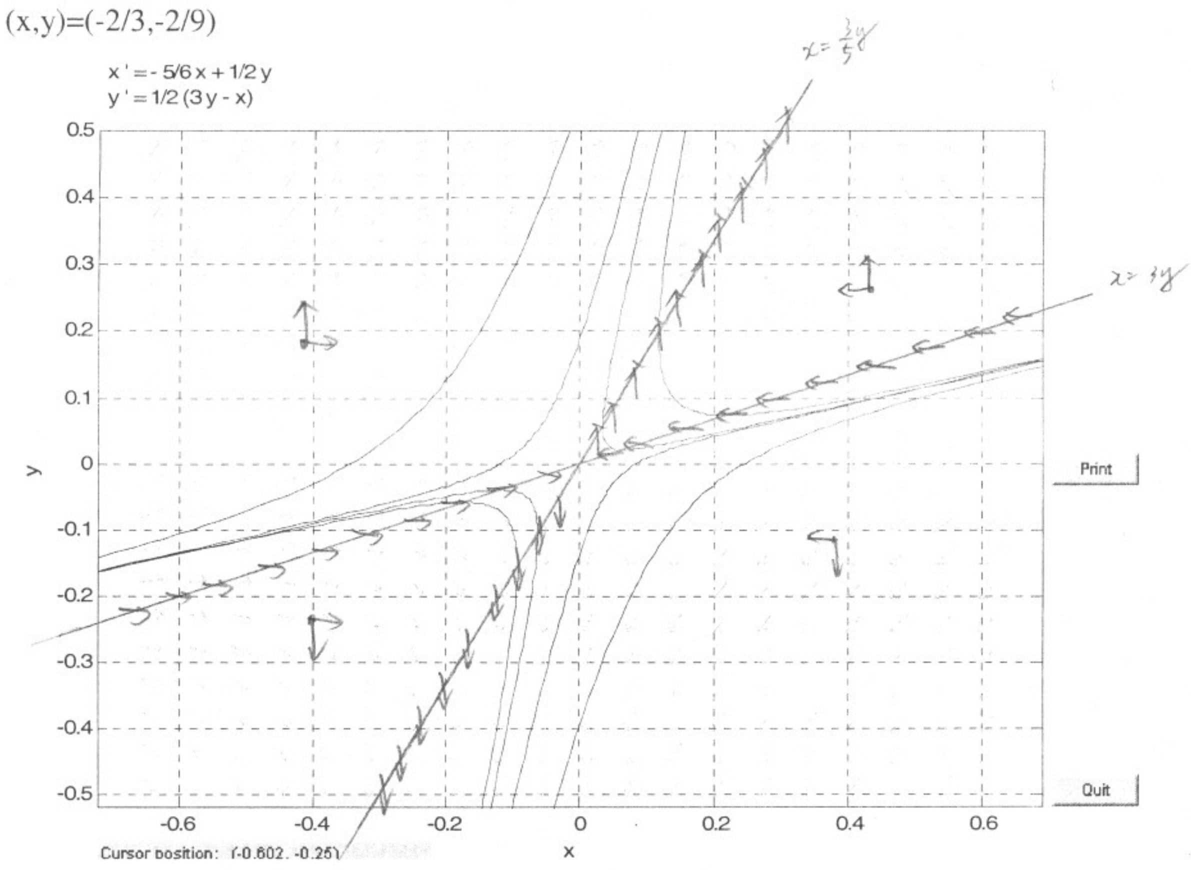
$\frac{dx}{dt} = \frac{1}{2}(3y+y)+9y^2$

$= 2y+9y^2 = 0$

$\frac{dx}{dt}>0$. $\frac{dx}{dt}>0$ $\frac{dx}{dt}>0$

$-\frac{2}{9}$ 0 y

(x,y)=(-2/3,-2/9)



x ' = - 5/6 x + 1/2 y
y ' = 1/2 (3 y - x)

fixed point $(x,y) = (\frac{-2}{3}, \frac{-2}{9})$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - \frac{4}{3} & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{5}{6} & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

ńu

$$J = \begin{pmatrix} -\frac{5}{6} & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

$$\beta = \text{tra } J = \frac{-5}{6} + \frac{3}{2} = \frac{-5}{6} + \frac{9}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\sigma = \det J = -\frac{5}{6} \cdot \frac{3}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{-5}{4} + \frac{1}{4} = -1$$

∴ saddle point ..

Nullclines = $x = \frac{3}{5}y$, $x = 3y$

$$\begin{cases} \frac{dx}{dt} = -\frac{5}{6}x + \frac{1}{2}y = 0 \\ \frac{dy}{dt} = -\frac{1}{2}x + \frac{3}{2}y = 0 \end{cases}$$

when $x = \frac{3}{5}y$, $\frac{dx}{dt} = 0$

$$\frac{dy}{dt} = -\frac{1}{2}x + \frac{3}{2}y = -\frac{3}{10}y + \frac{3}{2}y$$

$$= (-\frac{3}{10} + \frac{15}{10})y = \frac{12}{10}y$$

$\frac{dy}{dt} < 0$ $\frac{dy}{dt} > 0$

0

when $x = 3y$ $\frac{dy}{dt} = 0$

$$\frac{dx}{dt} = -\frac{5}{6}x + \frac{1}{2}y = -\frac{5}{\cancel{6}_{2}} \cdot \cancel{3}y + \frac{1}{2}y = -2y$$

$\frac{dx}{dt} > 0$ $\frac{dx}{dt} < 0$ y

0

in this case. the linear case is similar to

the nonlinear

After linearization
(x,y)=(0,0)

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x'=2x+4y
y'=-2y
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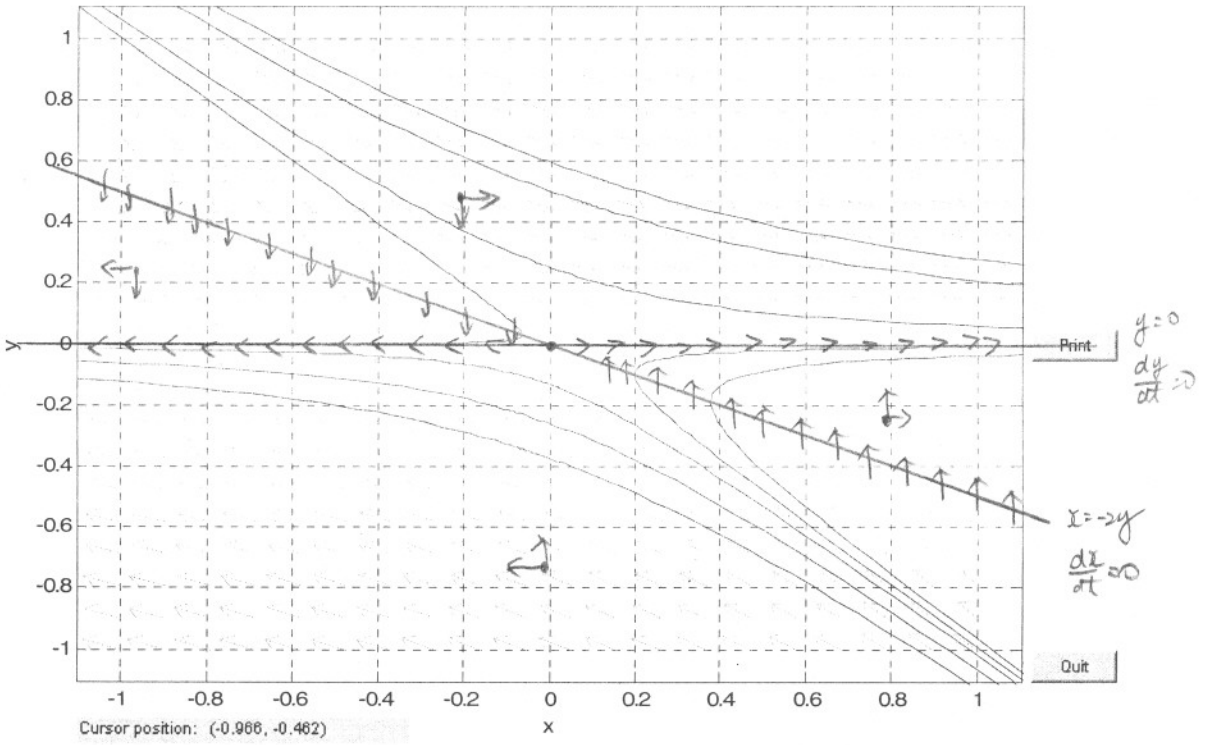


Cursor position: (-0.966, -0.462)

fixed point (x,y)=(0,0)

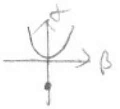
$$\frac{d}{dt}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2&4\\0&-2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$J=\begin{pmatrix}2&4\\0&-2\end{pmatrix}$$

✓

$\beta = \mathrm{tr}\,J = 0$

$\gamma = \det J = -4$



∴ SADDLE POINT #

$\lambda^2 - 4 = 0$

$\lambda = \pm 2$ #

Nullclines: $x=-2y$, $y=0$.

$$\begin{cases}\frac{dx}{dt} = 2x+4y = 0\\ \frac{dy}{dt} = -2y = 0\end{cases}$$

when $x=-2y$, $\frac{dx}{dt}=0$

$\frac{dy}{dt} = x$

$\frac{dy}{dt}<0 \quad \frac{dy}{dt}>0$, 0, x

when $y=0$, $\frac{dy}{dt}=0$

$\frac{dx}{dt} = 2x$

$\frac{dx}{dt}<0 \quad \frac{dx}{dt}>0$, 0, x

in this case, the linear and nonlinear phase protrait are similar.