Solution of Chemical Reaction Problem, 94/28

The solutions uses the **Phase Line**. The key is the sign of the function

\[ f(x) := \alpha(p - x)(q - x). \]

The concentrations \( p, q \) are unequal numbers belonging to the interval \([0, 1]\)
The parameter \( \alpha \) is positive.
Since \( f(x) \) is a quadratic polynomial, the graph of \( f(x) \) is the a parabola.
Since the coefficient of \( x^2 \) is equal to \( \alpha > 0 \), the parabola opens upward.
The parabola crosses the \( x \)-axis at the roots \( p, q \). Thus \( f(x) > 0 \) for \( x \) below the two roots
and for \( x \) above the two roots.
For \( x \) between the two roots, \( f(x) < 0 \).
Therefore, solutions to the left of the two roots increase. As \( t \to \infty \) they converge to
the lesser of the two roots. This solves part a, showing that the solution starting at 0,
converges as \( t \to \infty \), to the smaller of \( p \) and \( q \).
The lesser of the two roots is stable.

Solutions to the right of the two roots increase. They diverge to infinity (in finite time).
For part b, the same analysis applies except that the interval between the roots degenerates
to a single point. Still solutions starting to the left of the root, increase to the root as \( t \to \infty \). Solutions starting to the right diverge to infinity (in finite time).
The explicit solution by formula is by separation of variables. The case of part b is easier
as it does not require partial fractions for the integration.