Math 555, Fall 2002 Prof. J. Rauch

Homework 1. Due Tues Sept 10

Read Chapter 1 before class on Thursday Sept 5. There are two important pieces of background to buttress at the start of a complex variables course. They are ideas from the differential calculus of maps from the plane to itself, and complex numbers. The first chapter is devoted to the latter together with a bit of plane geometry. Much will likely be familiar. Otherwise read more slowly. The differential calculus will be the core of Tuesday's class while complex numbers will occupy us on Thursday.

The first problems of the first two assignments are from advanced calculus and serve to remind you how partial derivatives can be used to analyse the behavior of functions mapping the plane to itself. They also have a computational and geometric side which is true of much that we will do in this course. The matrix of partial derivatives is sometimes called the Jacobian matrix after Jacobi. Denoting it by J the key relation is that for points x, y near $\underline{x}, \underline{y}$ one has

$$(\Delta u, \Delta v) \approx J(\underline{x}, \underline{y})\,(\Delta x, \Delta y)\,.$$

This fundamental relation is the two dimensional analogue of

$$\Delta y \approx f'(\underline{x})\Delta x$$

which is the key relation in differntial calculus of functions of one variable.

1. For inversion in the unit circle, defined by

$$(x, y) \rightarrow (u, v) = \frac{(x, y)}{x^2 + y^2},$$

compute the Jacobian matrix

$$\begin{bmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{bmatrix}.$$

Use this to show that infinitesimal circles are sent to infinitesimal circles. That is a circle of radius $r << 1$ centered at a point $\underline{x}, \underline{y}$ is sent to a set which is very close (error $\sim r^2$) to a circle centered at the image of the central point. The images which might have been ellipses with any eccentricity, have eccentricity is equal to 1. If you find the analysis difficult you can check the result with computer simulations. (Plot the image of small circles, for example using Matlab). That may help you understand the assertion and thereby help with the computation.

The next 10 short problems are **2** through **11** Many of them are straight forward. In many of the multipart questions only the parts in parentheses are required. The other parts are often just as interesting.

5/1(b,d), 4(a,c) Warning: 4c requires a computation.

11-12/7(you may take $z_1 = 1$)

17-18/3(a), 5a, 7

22-23/2(b,d)

25/1 ,2, 3