1. i. Show that the function $1/\sin z$ has simple poles at the points $n\pi$ with residue equal to $\cos n\pi = (-1)^n$.

ii. Show that $\cos \pi z/\sin \pi z$ has poles at the integers $n$ with residues equal to $1/\pi$.

iii. Let $C_N$ be the circle of radius $N + 1/2$ with center equal to the origin. Show that

$$\lim_{N \to \infty} \oint_{C_N} \frac{1}{z^2} \frac{\cos \pi z}{\sin \pi z} \, dz = 0.$$ 

2. iv. Compute

$$\text{Res} \left( \frac{1}{z^2} \frac{\cos \pi z}{\sin \pi z}, 0 \right).$$

v. Apply the Residue Theorem to compute $\sum_{n=1}^{\infty} 1/n^2$.

**Discussion.** This is a broadly applicable method to sum infinite series using residues. It sums, for example, $\sum_{n=-\infty}^{\infty} P(n)/Q(n)$ for polynomials $P,Q$ so that

- $Q$ has no real roots,
- $\deg Q \geq \deg P + 2$, and,
- one knows exactly the roots of $Q$.

The problem is a case where $Q$ does have a root on real axis.

3. Use the result of problem 6 of Homework 7 whose solution is posted on my office door, to compute using antiderivatives,

$$\int_{0}^{\infty} \frac{x}{1 + x^3} \, dx.$$ 

**Discussion.** This is an example where the antiderivative works and the standard residue method does not. **Warning.** Be careful about branches of logarithms. You need them to be defined on the positive real axis.

4. Compute

$$\int_{0}^{\infty} \frac{x \sin 2x}{x^2 + 3} \, dx.$$ 

Ans. $(\pi/2)e^{-2\sqrt{3}}$.

**Discussion.** This integral is not absolutely convergent, but exists as an improper Riemann integral.

5. Evaluate using residues

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{x^3 + 1}{x^4 + 1} \, dx.$$
**Hint.** The example in class where the degree of $P$ is one lower that the degree of $Q$ is a model.

6. 188/15.

7. Exercise on Homework 1.