Homework 4. Due Oct. 3

103/2, 9, 10, 13

120/4 (You need to explain why the hypotheses are verified)

120/5

120/12 Alternate Hint: The hint is Green's Theorem.

120/13

9. Suppose that $F = \big(F_x(x,y), F_y(x,y)\big)$ is a continuously differentiable vector valued function on a nice closed bounded domain R in the plane. Derive the planar Divergence Theorem by applying Green's Theorem to the flux integral

$$\oint_{\partial R} F.N\, ds := \oint_{\partial R} F_x dy - F_y dx\,.$$

Discussion. If $(x(t), y(t))$ is a parametrization of positively oriented part of the boundary, then the unit tangent vector is given by

$$\frac{(dx/dt\,,\, dy/dt)}{((dx/dt)^2 + dy/dt)^2)^{1/2}}\,.$$

The two unit normal vectors are then given by

$$\pm\ \frac{(dy/dt\,,\, -dx/dt)}{\big((dx/dt)^2 + (dy/dt)^2\big)^{1/2}}\,.$$

The inward normal is so that the matrix with T as first row and N as second row has positive determinant. Therefore the outward normal yields a negative determinant so the outward normal is given by the above formula with the plus sign. Therefore

$$F \cdot N = \Big(F_x\,\frac{dy}{dt} - F_y\,\frac{dx}{dt}\Big)/\big((dx/dt)^2 + (dy/dt)^2\big)^{1/2}\,.$$

Since $ds = \big((dx/dt)^2 + (dy/dt)^2\big)^{1/2} dt$, this explains why $F \cdot N\, ds = F_x\, dy - F_y\, dx$ yielding the stated equivalence between the ds and the dx, dy versions of the flux integrals. The latter are prefered.

10. Derive the planar Stokes Theorem by applying Green's Theorem to the circulation integral

$$\oint_{\partial R} F.T\, ds = \oint_{\partial R} F_x dx + F_y dy\,.$$