Math 555, Fall 2002 Prof. J. Rauch

Homework 5. Due Oct. 10

1. Use the power series

$$e^z = \sum_0^\infty \frac{z^n}{n!}$$

to verify by direct substitution and rearrangement that

$$e^{z+w} = e^z\, e^w\,.$$

Discussion. The rearrangements are justified because the series are absolutely convergent. Inside the disc of convergence, power series are always absolutely convergent. If *Proof.* $\sum a_n z^n$ converges for a $\underline{z}$ with $|\underline{z}| = R$, then the terms of that series are bounded so

$$|a_n\, \underline{z}^n| \;=\; |a_n R^n| \;\le\; M \qquad \text{so} \qquad |a_n| \;\le\; M/R^n\,.$$

Then for $|z| < R$,

$$\sum |a_n z^n| \;=\; \sum |a_n\, \underline{z}^n (z/\underline{z})^n| \;\le\; \sum M(|z|/R)^n \;=\; \frac{M}{1-|z|/R}\,. \qquad \text{q.e.d.}$$

The Weierstrass M-Test shows that this computation proves the stronger result that for $r < R$ the series converges absolutely and uniformly in $|z| \le r$.

128-29/2,3,4,9

136/1,2,4,5,6

Midterm Exam Thursday Oct 17. In class. You may have one side of a 3in.$\times$ 5in. card of notes from home. Writing small you will find that you can fit all the facts you'd like on such a premitted crib sheet. Save the card for the final where you will be allowed a second side.

The exam immediately follows the study break which strikes me as poetic justice.

I will be available to answer questions during study break only by email as I'll be out of town. I will sign on. Will be in town till Friday Oct 11. Will miss office hours on the study day Tuesday Oct 15.

Homework problems are a good model for what will be on the exam. You will be expected to understand the basic definitions and results of the course and how they can be used. The exam will not emphasize proof. There might be one proofish item max.