

Math 558. Advanced Ordinary Differential Equations and Dynamical Systems
Fall 2009

Prof. J. Rauch

Prerequisites. Basic Linear Algebra, Ordinary Differential Equations (Math 216), Multivariable Calculus (215). Some exposure to more advanced mathematics *e.g.* Advanced Calculus (math 450/451) or Advanced Mathematical Methods (Math 454).

Text. F. Brauer and J. Nohel, *The Qualitative Theory of Ordinary Differential Equations: An Introduction*, Dover Publications.

Homework. Graded assignments regularly.

Exams. In class midterm and final.

Course Description.

Differential equations arise in the mathematical description of numerous systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the modern (post Poincaré) theory. The course surveys a broad range of topics with emphasis on techniques, proofs, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program.

Outline.

- Causality and the existence and uniqueness of solutions.
- Linearization at equilibria and constant coefficient systems. (The case of multiple eigenvalues and the ergodic properties of coupled oscillators complement introductory treatments).
- The geometry of phase plane trajectories. Lyapunov's method. LaSalle's invariance principle. Conjugation of sinks/sources. Stable and unstable manifolds. Gradient flow. Introduction to hamiltonian systems.
- Periodic solutions, stability, Poincaré map, ω -limit set, Poincaré-Bendixson, forced oscillations, and dynamical systems.
- Introduction to bifurcation theory of equilibria. Pitchfork and Hopf.
- Introduction to chaotic dynamics. Definitions and first examples.

Grading. Grades are based on Homework, Midterm Exam, and a Final Exam. The relative weighting will depend on the availability of graders. If there is good grader support, homeworks will count more.