Math 558. Advanced Ordinary Differential Equations and Dynamical Systems  
Fall 2010  
Prof. J. Rauch  

**Prerequisites.** Basic Linear Algebra, Ordinary Differential Equations (Math 216), Multivariable Calculus (215). Some exposure to more advanced mathematics *e.g.* Advanced Calculus (math 450/451) or Advanced Mathematical Methods (Math 454).

**Text.** M. Hirsh, S. Smale, and R. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, 2nd ed., Elsevier. *n.b.* The library has an electronic version of this text which can be freely consulted.

**Guest Professor.** The first two meetings of this course will be conducted by Professor P. Federbush.

**Homework.** Graded assignments weekly.

**Exams.** In class midterm plus final exam.

**Course Description.** Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the modern (post Poincaré) theory. The course surveys a broad range of topics with emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Proofs are given. Homeworks and exam concentrate on using rather than proving the results.

**Outline.**
- Phase line. Dynamics in dimension 1 and 1.5. Bifurcations. Poincaré map.
- Existence, uniqueness and and perturbations.
- Theory of linear systems.
- The geometry of phase plane of linear systems.
- Stable and unstable manifolds.
- Conjugation of sinks/sources.
- Lyapunov’s method. LaSalle’s invariance principal.
- Gradient flows and hamiltonian systems.
- Periodic solutions. Poincaré map, ω-limit set, Poincaré-Bendixson, forced oscillations.
- Bifurcation theory of equilibria. Pitchfork and Hopf.
- Introduction to chaotic dynamics. Definitions and first examples.
- Selected applications.

**Grading.** Homework 35%, Midterm Exam 25%, and Final Exam 40%.