1. 135/1h.

2. 136/7. **Discussion.** Here is what is meant by part d. The general solution will have families of solutions periodic with two periods. In the spirit of §6.2 show that under suitable conditions on the periods, all solutions are periodic. And when those conditions are violated, solutions are typically not periodic.

3. 137/9.

4. Prove the **Cayley-Hamilton Theorem.** If $A$ is an $N \times N$ matrix and $p(z) = \det(zI - A)$ is its characteristic polynomial, then $p(A) = 0$.

**Remarks.** i. Since $A^j$ is an $N \times N$ matrix, $p(A)$ is a sum of such matrices. ii. We proved the two by two case in class. iii. The suggestion of the next hint is a better proof relying on generalized eigenspaces which we did not have at the time. **Hint.** Factor $p(z) = \prod_{j=1}^k (z - \lambda_j)^{m_j}$ with distinct $\lambda_j$. Then $p(A) = \prod_{j=1}^k (A - \lambda_j I)^{m_j}$. Use the Spectral Decomposition Theorem to show that $p(A)v = 0$ for each vector $v$ in a generalized eigenspace $X_\mu$. Then represent an arbitrary vector as a sum of these.

5. Consider

$$A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

from the example on page 130 of Hirsch-Smale-Devaney. Write in the form of an integral using the formula of variation of parameters the solution of

$$X' = AX + (\sin t, 0), \quad X(0) = 0.$$

**Remark.** The method of variation of parameters is very useful in analysis and not so useful in computing. In this exercise, the integrals can be computed. If the forcing term had been $(e^{t^2}, 0)$ the integrals could not be computed explicitly.

6. The first exercise of §9 of the freshly modified Dimension 1.5 posting.