1. Describe all bifurcations and the global bifurcation diagram for the scalar periodic equation depending on the real parameter $a$,

$$\theta' = \sin^2 \theta + a.$$ 

2. 186/8. **Hint.** Use the preceding problem.

3. The new Exercise 9.1 of the Dimension 1.5 handout.

4. Find all equilibrium solutions of the following system and determine, where possible, their stability by linearization,

$$x' = x^2 + y^2 - 1, \quad y' = 2xy.$$ 

If the linearization is a saddle, find the tangent line at the origin to the stable manifold.

5. Find all equilibrium solutions of the following system and determine, where possible, their stability by linearization,

$$x' = x^2 + y^2 - 1, \quad y' = x^2 - y^2.$$ 

If the linearization is a saddle, find the tangent at the origin to the stable manifold.

6. The new Exercise 1.1 in the Generalized Eigenspace handout.

7. Show that

$$x' = \ln(1 - z), \quad y' = \ln(1 - x), \quad z' = \ln(1 - y),$$

has only one equilibrium and that it is hyperbolic (no purely imaginary eigenvalues for the linearization). Find the dimensions of the stable and unstable manifolds. Find the tangent space at the equilibrium of the stable and unstable manifold. **Hint.** You may use the result of Exercise 1.2 in the Generalized Eigenspace handout. **Remark.** That exercise will be on the next assignment.

8. Determine the stability of the equilibrium 0 of the following system from its linearization,

$$x_1' = -2x_1 + x_2 + 3x_3 + 9x_2^3$$

$$x_2' = -6x_2 - 5x_3 + 7x_3^5$$

$$x_3' = -x_3 + x_1^2 + x_2^2.$$