1. **Nonlinear friction continued.** Consider free fall with nonlinear friction as in Homeworks 3,4.
   i. Find the asymptotic behavior as $t \to \infty$ of the approximate solution found in Homework 3.
   **Discussion.** This divergence to infinity is called secular growth.
   ii. Does the asymptotic behavior of the approximate solution predict that of the exact solution as computed in Homework 4.
   **Discussion.** The approximate solution is guaranteed to be a good approximation on bounded time intervals.
   iii. Show that $\lim_{\varepsilon \to 0} \lim_{t \to \infty} v_{\text{approx}} \neq \lim_{t \to \infty} \lim_{\varepsilon \to 0} v_{\text{approx}}$.
   **Discussion.** i. It is normal that an approximation has small errors. Those errors accumulate over time. It is rare that an approximate solution gives a good approximation for very large times. ii. Concerning the third part V. Arnold offers the excellent example of a bucket full of water with a hole in the bottom of size $\varepsilon$. For that system, $\lim_{\varepsilon \to 0} \lim_{t \to \infty}$ is an empty bucket while $\lim_{t \to \infty} \lim_{\varepsilon \to 0}$ is a full bucket.

2. Exercise 1.1 of the Spectral Decomposition Handout.

3. Show that if the $2 \times 2$ matrix $A$ has only one eigenvalue $\lambda$ and there is a basis of eigenvectors, then $A = \lambda I$.

4. 71/1.

The next problems use the Spectral Decomposition Theorem to find the general solution of $X' = AX$ when $A$ need not have a basis of eigenvectors. The method is succinctly described in the Multiple Root Algorithm handout.

5. 135/1a.

6. 135/5.

7. 135/1c.