

Math 558 Fall 2006
Homework 1
Due September 21

There are four problems and one challenge. Three are fairly straight forward. The fourth requires you to construct an easy rigorous proof. The goal of the proof problems will be to increase your skills. This first one will serve to find out where we stand now. I understand that many of you may have little experience of this sort.

1. 16/4. This means page 16 problem 4 of the text.
2. 16/13.
3. This problem shows that the only possible limits as $t \rightarrow \infty$ of solutions of

$$\frac{dx}{dt} = f(x), \quad f \in C^1(\mathbb{R}),$$

are equilibria. Prove that if $x : [0, \infty[\rightarrow \mathbb{R}$ is a solution and

$$\lim_{t \rightarrow \infty} x(t) = x_0, \tag{1}$$

then $f(x_0) = 0$. **Hints.** Want to show that (1) and $f(x_0) \neq 0$ is impossible. If $f(x_0) \neq 0$, show that there is a $\delta > 0$ so that $|f(x)| > |f(x_0)|/2$ for all x so that $|x - x_0| \leq \delta$. Choose $T > 0$ so that

$$t \geq T \implies |x(t) - x_0| \leq \delta.$$

Show that these two things cannot both occur. Conclude that $f(x_0) \neq 0$ together with (1) is impossible. That is the desired conclusion.

Challenge. 157/9. **Hint.** The previous hint and discussion referred to problem 8. Sorry.