Problems for 1 – d Mechanical Systems

1 Potentials with one critical point.

Each of the following six graphs is the graph of a potential $V(x)$ with exactly one critical point. You may assume that $V'' \neq 0$ at the critical points. You may also assume that for large $x$ either $V$ tends to infinity or a horizontal asymptote as indicated in the figure. In each case you are asked to sketch the entire phase portrait of the mechanical system

$$x'' = -\frac{\partial V(x)}{\partial x}.$$
2 Potentials with two critical points.

Perform the same analysis for the following potentials each of which has two critical points.
3 Potentials with three critical points.

Perform the same analysis for the following potentials each of which has three critical points.