Meet the Laplacian. Spherical Symmetry and the Mean Value Property

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Outline

• Rotational invariance of ∆ and harmonic functions.
• Symmetry derivation of MVP
• The Laplacian as Infinitesimal MVP
• Equal angles of isosceles triangle, symmetry proof from Euclid’s elements
• Coulomb’s Law and Gauss’ Law
• Symmetry proof of Newton’s Theorem
• Proof of Newton’s Theorem by MVP
• The five point Laplacian
• Discrete Dirichlet Problem
• Random walk and the discrete Dirichlet problem
• Random walk and the Dirichlet problem.

The Coulomb, Gauss, Newton trilogy is an introduction to Electrostatics and prepares the Meet Electrostatics topics.

Course Materials. There is a handout on the Infinitesimal MVP. There a handout on Haar measure. One on Coulomb, Gauss, and Newton. The last three topics are inspired by the classic paper of Courant, Friedrichs, and Lewy, a translation of which is posted on my home page. Click to course materials and scroll to Encounters with Partial Differential Equations.

A set of Exercises on the laplacian and symmetry is also my home page. It contains yet another proof of the Mean Value Property that requires identifying all orthogonally invariant polynomials.